

2.1

Two-Term and Three-Term Ratios

Focus on...

After this lesson, you will be able to...

- represent two-term and three-term ratios
- identify, describe, and record ratios from real-life examples
- represent a ratio as a fraction or as a percent
- solve problems using ratios



You use ratios to enlarge or reduce graphics and pictures. Discuss with a partner how you could use ratios to describe these two images.

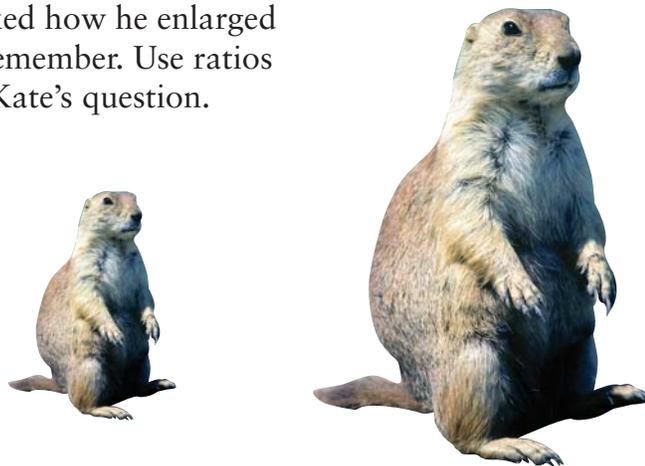
Explore the Math

Materials

- ruler

How can you compare an enlargement to its original image?

Jason made the following enlargement of a prairie dog. Kate asked how he enlarged it. Jason could not remember. Use ratios to help him answer Kate's question.



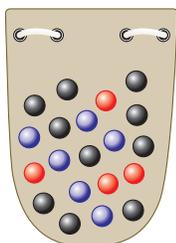
1. a) How might you compare the original photo to its enlargement?
Share your method with a classmate.
 - b) What might you compare on the two photos to give you a ratio?
 - c) How many measurements would you need to make?
 - d) Why would it be important to measure exactly the same parts of each photo?
2. Suggest a multiplier that Jason would have used to enlarge the photo of the prairie dog. How did you arrive at your answer?
Compare your answer with those of your classmates.

Reflect on Your Findings

3. a) If the ratio comparing the image to the original is greater than one, what does this tell you?
- b) If the ratio comparing the image to the original is less than one, what does this tell you?

Example 1: Represent Ratios

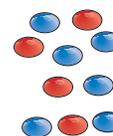
A bag contains 20 marbles.



- a) What is the **two-term ratio** of black to red marbles?
- b) Compare the number of red marbles to the total number of marbles. Write the ratio as an equivalent fraction in lowest terms.
- c) What marbles are represented by the ratio 6:10?
- d) Write the **three-term ratio** comparing the red, purple, and black marbles.

two-term ratio

- compares two quantities measured in the same units
- written as $a:b$ or a to b



blue: red is 6:4

three-term ratio

- compares three quantities measured in the same units
- written as $a:b:c$ or a to b to c



blue:red:brown is 6:4:2

Strategies

Draw a Diagram

part-to-part ratio

- compares different parts of a group to each other
- 10:8 is the part-to-part ratio of brown to red beads.



part-to-whole ratio

- compares one part of a group to the whole group
- 5:23 is the part-to-whole ratio of blue to total number of beads.



Solution

a) Method 1: Represent a Ratio Using a Drawing

There are 10 black and 4 red marbles. The drawing shows the ratio.



This is a **part-to-part** ratio.

Method 2: Represent a Ratio Using Symbols or Words

The ratio of black marbles to red marbles is 10:4 or 10 to 4.

- b)** There are 4 red marbles out of a total of 20 marbles. The ratio can be expressed as 4:20.

A **part-to-whole ratio** can be expressed as a fraction.

$$\frac{\text{red}}{\text{total}} \text{ is } \frac{4}{20}$$

Write the fraction as an equivalent fraction in lowest terms.

$$\frac{4}{20} = \frac{1}{5}$$

To express a fraction in lowest terms, divide the numerator and the denominator by the same number.

- c)** There are 6 purple marbles and 10 black marbles. purple:black is 6:10

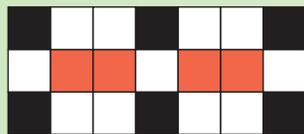
- d)** You can compare red, purple, and black marbles using a three-term ratio.

$$\begin{aligned} \text{red:purple:black} &= 4:6:10 \\ &= 2:3:5 \end{aligned}$$

You can multiply or divide each term of a three-term ratio by the same number.

Show You Know

Use the design to help answer the following questions.



- What is the ratio of red tiles to total tiles? Express the answer three different ways.
- What could the ratio 4:6 represent?
- What is the ratio of red to black to white tiles?

Example 2: Apply Ratios

Tamara has a recipe for fruit punch that calls for three cans of frozen orange juice concentrate, two cans of raspberry juice concentrate, and one can of lime juice concentrate. For each can of juice concentrate, the directions say to add three cans of water. All the cans are the same size. Tamara makes one recipe of fruit punch.

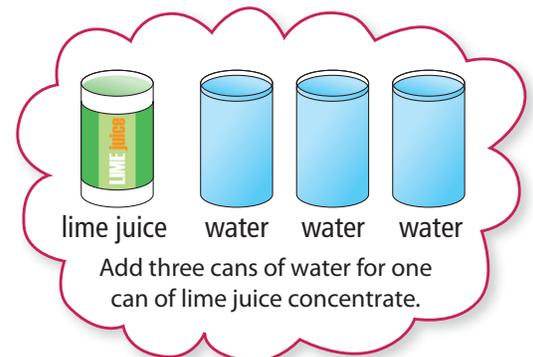
a) Copy and complete the following chart.

	Orange	Raspberry	Lime	Total
Juice Concentrate (cans)				
Water (cans)				
Total Punch (cans)				

- b) What is the ratio of orange juice to lime juice concentrate?
Express the ratio two different ways.
- c) What is the ratio of lime to orange to raspberry juice concentrate?
- d) What is the ratio of water to juice concentrate?
- e) How many cans of punch does the recipe make?
- f) What is the ratio of orange, raspberry, and lime juice concentrate to total punch? Express the ratio as a fraction, a decimal, and a percent.

Solution

	Orange	Raspberry	Lime	Total
Juice Concentrate (cans)	3	2	1	6
Water (cans)	$3 \times 3 = 9$	$2 \times 3 = 6$	$1 \times 3 = 3$	18
Total Punch (cans)				24



- b) three cans of orange juice and one can of lime juice concentrate
The ratio of orange to lime juice concentrate is 3 : 1 or 3 to 1.
- c) one can of lime juice, three cans of orange juice, and two cans of raspberry juice concentrate
The ratio of lime to orange to raspberry juice concentrate is 1 : 3 : 2.
- d) 18 cans of water, 6 cans of juice concentrate
The ratio of water to juice concentrate is 18 : 6 or 3 : 1.
- e) 18 cans of water + 6 cans of juice concentrate = 24 cans of punch
One recipe makes 24 cans of punch.

This is a part-to-part ratio.

$$\frac{18}{6} = \frac{3}{1}$$

÷ 6

f) 6 cans of juice concentrate, 24 cans of punch

$$\begin{array}{l} \text{juice concentrate} \\ \text{punch} \end{array} \text{ is } \frac{6}{24} = \frac{1}{4}$$

$\div 6$
 $\div 6$

$$\frac{1}{4} = 0.25$$
$$0.25 = 25\%$$

To convert the decimal to a percent, multiply by 100 and add the percent symbol.

The ratio of juice concentrate to punch is $\frac{1}{4}$, 0.25, or 25%.

Show You Know

A recipe for trail mix calls for three cups of mini pretzels, two cups of roasted soy chips, one cup of raisins, and one cup of sunflower seeds. You make two batches of trail mix.

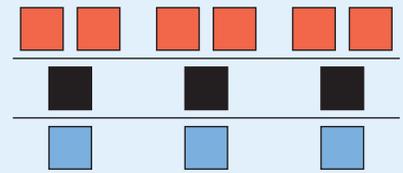
- What is the ratio of mini pretzels to raisins? Express the ratio two different ways.
- What is the ratio of roasted soy chips to sunflower seeds?
- How many cups of mix do two batches make?
- What is the ratio of soy chips and sunflower seeds to total trail mix? Express the ratio as a fraction, a decimal, and a percent.

Key Ideas

- A part-to-part ratio compares different parts of a group.
The ratio of red to black tiles is 6:3 or 6 to 3.
The ratio in lowest terms is 2:1 or 2 to 1.
- A part-to-whole ratio compares one part of a group to the whole group.
The ratio of red to total tiles is 6:12 or 6 to 12. The ratio in lowest terms is 1:2 or 1 to 2. One out of every two tiles is red.
A part-to-whole ratio can be written as a fraction, a decimal, and a percent.
The ratio of $\frac{\text{red}}{\text{total}}$ is $\frac{6}{12}$ or $\frac{1}{2}$, 0.5, 50%.

- A three-term ratio compares three quantities measured in the same units.

The ratio of red to black to blue tiles can be written as 6:3:3 or 6 to 3 to 3. The ratio in lowest terms is 2:1:1 or 2 to 1 to 1.

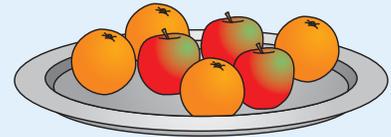


- A two-term ratio compares two quantities measured in the same units.

The ratio of black to total tiles can be written as 3:12 or 3 to 12. The ratio in lowest terms is 1:4 or 1 to 4. One out of every four tiles is black.

Communicate the Ideas

1. Janine wants to write the ratio of oranges to apples. How does she know whether to write 3:4 or 4:3?
2. Your friend missed the class when ratios were introduced. Use an example and draw a diagram to explain the difference between a part-to-part ratio and a part-to-whole ratio.
3. Give two examples of how ratios are used in daily life. Share your ideas with a classmate.
4. The fraction $\frac{2}{5}$ can be interpreted as two parts out of a total of five parts. Use a diagram to show an example of this part-to-whole ratio.



Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on pages 47–48.

5. Write each ratio using ratio notation. Do not write the answers in lowest terms.
 - a) \$2 compared to \$8.
 - b) The width of the cover of this book compared to its length, in centimetres.
 - c) In a class, 14 of 30 students are girls. What is the ratio of boys to girls to total students?
 - d) Your age compared to that of a 28-year-old person.
6. Write each ratio in #5 as an equivalent ratio in lowest terms.
7. Write each ratio in fraction form. Do not write the answers in lowest terms.
 - a) You spend \$4 out of \$10.
 - b) A team won three games and lost six games. What is the ratio of games won to games played?
 - c) A bag contains 12 red and 3 blue beads. Compare blue beads to total beads.
 - d) A pond contains 27 guppies and 33 goldfish. What is the ratio of guppies to total fish?

8. Identify the missing number to make an equivalent fraction.

a) $\frac{1}{2} = \frac{\blacksquare}{8}$

b) $\frac{4}{5} = \frac{12}{\blacksquare}$

c) $\frac{2}{7} = \frac{\blacksquare}{21}$

d) $\frac{\blacksquare}{4} = \frac{3}{12}$

e) $\frac{21}{49} = \frac{3}{\blacksquare}$

f) $\frac{4}{\blacksquare} = \frac{12}{15}$

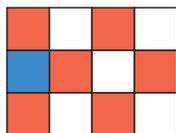
For help with #9 to #11, refer to Example 2 on pages 49–50.

9. Use the data about wins and losses on school teams to answer the questions.

Sport	Wins	Losses
Hockey	9	6
Volleyball	10	5
Baseball	12	8

- Which sports have equivalent win–loss ratios? Show how you know.
 - What is the ratio of wins to total games played for hockey? Give your answer as a fraction, a decimal, and a percent.
10. Tyler counted 20 cars in the school parking lot. Of these, 6 were red, 4 were green, and 1 was yellow.
- Draw a diagram to represent the situation.
 - How many cars were not red, green, or yellow?
 - What is the ratio of yellow to green to red cars?
 - What is the ratio of red to total cars? Express the ratio as a fraction and a percent.

11. What tiles could be represented by each of the following ratios?

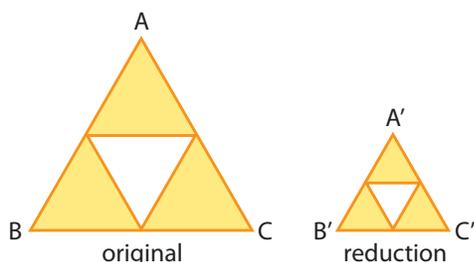


- 1 to 5
- $1:6:5$
- $\frac{1}{2}$
- $\frac{11}{12}$

Apply

12. In a class of 32 students, there are 24 girls.
- What is the boys to total students ratio? Express the ratio as a fraction and a percent.
 - What is the girls to boys ratio? Use ratio notation to express the ratio.
13. A soccer team played 28 games and won 4 out of every 7 games. There were no tied games.
- How many games did they lose?
 - What was the team’s win–loss ratio? Explain how you got your answer.
 - If this trend continues, how many losses would you expect the team to have once they have won 20 games?
14. Three eighths of the 96 adults in the McGregor clan are less than 150 cm tall.
- Draw a diagram to represent the statement.
 - How many adults in the McGregor clan are less than 150 cm tall? Show your work using equivalent ratios.
 - How many adults are 150 cm or taller? Explain your thinking.
15. Diana and John are making three-cheese lasagna. The recipe calls for 100 g of Romano, 300 g of mozzarella, and 250 g of cottage cheese.
- Write a ratio in lowest terms to compare the amounts of the three cheeses. State the order of the cheeses.
 - What amounts of Romano and cottage cheese do you need to make lasagna that contains 900 g of mozzarella cheese? Hint: Use equivalent ratios to help you.

16. Heather used a copier to make the following 50% reduction.



- Write the ratio of the length of $A'B'$ compared to the length of AB .
- Write the ratio of the length of $A'C'$ compared to the length of AC .
- Use your knowledge of ratio and percent to explain the meaning of a 50% reduction.

Literacy Link

A' is read as "A prime."
 A' labels the point in the reduction that corresponds to point A .

17. There are 48 passengers on a transit bus. At the next stop, 16 passengers got off and 12 others got on the bus.
- What is the ratio of the passengers who got off the bus compared to the original number on the bus? Show the ratio in lowest terms.
 - What is the ratio of the passengers who got on the bus at the stop compared to the new total then on the bus? Write your answer as a fraction, a decimal, and a percent.

18. The ratio of the width to the length of the Canadian flag is 1 : 2.



- The flag on the cover of an atlas is 12 cm wide. How long is it?
- A large flag outside a Calgary school is 3 m long. What is its width?

WWW Web Link

To practise solving ratio problems, go to www.mathlinks8.ca and follow the links.

19. The table gives the lengths of some rivers in Western Canada.

River	Length (km)
Churchill	1608
Fraser	1368
MacKenzie	1800
North Saskatchewan	1392
Thelon	904

- Write a ratio in fractional form to compare the length of the Churchill River and the Mackenzie River. Express the ratio as a fraction in lowest terms.
 - Write a ratio in decimal form to compare the length of the North Saskatchewan River and the Fraser River. Express the ratio as a decimal correct to the nearest hundredth.
 - The calculator keying sequence that compares the length of the Thelon River and the Churchill River is $\boxed{C} \boxed{904} \boxed{\div} \boxed{1608} \boxed{=}$ 0.5621890547. Write the decimal to the nearest hundredth. What comparison statement can you make about the length of the two rivers?
20. A 30-kg bag of fertilizer is labelled 15–20–10. This means that it contains 15% nitrogen, 20% phosphorus, and 10% potassium by weight. How many kilograms of nitrogen, phosphorus, and potassium are in the bag?

Did You Know?

Fertilizer ratios indicate the percent of nitrogen, phosphorus, and potassium. This bag of fertilizer contains 15% nitrogen, 20% phosphorus, and 10% potassium by weight. The remaining 55% is made up of other micronutrients and filler.



Extend

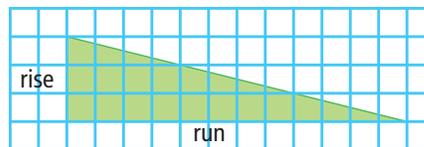
21. A golden rectangle has a length to width ratio called the golden ratio, which is approximately 1.62.
- Which of the following dimensions of rectangles are examples of golden rectangles?
 - $24 \text{ m} \times 38.9 \text{ m}$
 - $52 \text{ cm} \times 120.5 \text{ cm}$
 - $348 \text{ mm} \times 565 \text{ mm}$
 - If the width of a golden rectangle is 6.4 m, what is its length? Give your answer to the nearest tenth of a metre.

Did You Know?

The golden rectangle is used often in art and architecture. For example, the front of the Parthenon, a temple in Athens, Greece fits into a golden rectangle.



22. The side view of a ramp is shown.



- Express the ratio of rise to run in lowest terms. This ratio describes the slope of the ramp.
- Express the slope ratio as a fraction, a decimal, and a percent.
- Predict what effect each of the following would have on the slope of the ramp:
 - increasing the rise
 - decreasing the rise
 - increasing the run
 - decreasing the run

Did You Know?

The ratio $\frac{\text{rise}}{\text{run}}$ is called *slope*. Slope describes the steepness of roads, ramps, and ski runs.

Web Link

For more information about the golden ratio in nature, architecture, art, poetry, and music, go to www.mathlinks8.ca and follow the links.

MATH LINK

Plan an invitation for your international meal. Create a logo as part of the front of the invitation. A logo is an identifying symbol used in advertising.

- Design your logo using colours or measurements to show each of the following ratios

4:3 2:3:4

For example, if you use a rectangle in your logo, you could show that the length to width ratio is 4:3.

- Draw the logo on a 36 cm^2 section of centimetre grid paper.
- Identify the ratios used in your logo.

2.2

Rates

Focus on...

After this lesson, you will be able to...

- express rates using words and symbols
- identify, describe, and record rates from real-life examples
- solve problems using rates



Trainers use technology to accurately and reliably monitor the heart rate of an equine competitor. Measuring the heart rate helps evaluate a horse's physical condition. The heart rate can be read at rest, during exercise, or during recovery after an event.

Heart rate is measured by counting the number of beats per minute. Note that a **rate** has two units. The units for heart rate are beats and minutes. Other common rates include growth rates and fuel efficiency rates. For example, a plant may grow 6 cm per month, and the fuel efficiency for a specific vehicle may be 6.8 L per 100 km.

What are some other rates you know about? What units are commonly used to measure these rates?

rate

- compares two quantities measured in different units
- \$1.69 per 100 g or \$1.69/100 g is a rate for purchasing bulk food
- 72 beats per minute or 72 beats/min is a heart rate

Materials

- standard paper clips
- jumbo paper clips

Explore the Math

How can you determine a conversion rate?

Work with a partner. You will need a chain of standard paper clips and a chain of jumbo paper clips.

1. Use the paper clip chains to measure the lengths of six different objects in the classroom. Record your data.



Did You Know?

Paper clips come in various sizes. In Canada, standard paper clips are about 33 mm in length. Jumbo clips are about 50 mm in length.

unit rate

- a rate in which the second term is one
- for example, 20 km/h and 64 beats/min

Did You Know?

Speed is a unit rate that compares the distance travelled to the time it takes.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Speed is often measured in kilometres per hour or abbreviated as km/h.

2. What two units of measure are you using?
3. How can you use your data to determine a multiplier that describes the number of standard paper clips to one jumbo clip? This multiplier is called a conversion rate.

Reflect on Your Findings

4. a) A conversion rate is sometimes called a **unit rate**. Explain why.
b) Would the conversion rate for the number of jumbo clips for one standard clip be greater or less than one? Explain your thinking.
c) Is the conversion rate between one jumbo clip and one standard clip always the same? Why or why not?

Example 1: Determine Unit Rates

Ruby-throated hummingbirds and monarch butterflies travel similar paths across the Gulf of Mexico. The distance is just over 800 km. It takes the hummingbird 18.5 h and the monarch butterfly 41.6 h to cross the Gulf.



- a) Estimate the speed of the hummingbird and the butterfly.
- b) Calculate the speed of the hummingbird and the butterfly. Give each answer to the nearest hundredth.

Solution

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

	Hummingbird	Butterfly
a) Estimate speed.	$\frac{800 \text{ km}}{20 \text{ h}} = 40 \text{ km/h}$	$\frac{800 \text{ km}}{40 \text{ h}} = 20 \text{ km/h}$
b) Calculate speed.	$\frac{800 \text{ km}}{18.5 \text{ h}}$ $\text{C } 800 \div 18.5 \equiv 43.243243$ The speed is 43.24 km/h.	$\frac{800 \text{ km}}{41.6 \text{ h}}$ $\text{C } 800 \div 41.6 \equiv 19.230769$ The speed is 19.23 km/h.

Strategies
Estimate and Check

A rate can be expressed as a fraction that includes the two different units. A rate cannot be expressed as a percent because a percent is a ratio that compares quantities expressed in the same units.

The speed of the hummingbird is 43.24 km/h and the speed of the monarch butterfly is 19.23 km/h, to the nearest hundredth. The estimates suggest that these answers are reasonable.

Show You Know

Determine the unit rate in each situation.

- a) Brandon runs 150 m in 25 s.
- b) Kira earns \$88 for working 8 h.
- c) Cat food costs \$9 for five cans.

Example 2: Compare Prices Using Unit Rates

Brett went to the grocery store to buy his favourite brand of orange juice. He found the following container sizes and prices. Which container of orange juice is the best buy?



unit price

- a unit rate used when shopping
- often shown per 100 g or per 100 mL
- makes it easier for shoppers to compare costs of similar items



Solution

Calculate the **unit price** of each container of orange juice and then compare.

414 mL for \$1.69

$$\begin{aligned}\text{Unit price} &= \frac{\text{cost}}{\text{volume}} \\ &= \frac{\$1.69}{414 \text{ mL}} \\ &= \$0.00408/\text{mL}\end{aligned}$$

$$\text{C } 1.69 \div 414 \equiv 0.0040821$$

The unit price is \$0.00408/mL or 0.408¢/mL.

946 mL for \$2.99

$$\begin{aligned}\text{Unit price} &= \frac{\text{cost}}{\text{volume}} \\ &= \frac{\$2.99}{946 \text{ mL}} \\ &= \$0.00316/\text{mL}\end{aligned}$$

$$\text{C } 2.99 \div 946 \equiv 0.0031607$$

The unit price is \$0.00316/mL or 0.316¢/mL.

1.89 L for \$5.49

To compare unit prices, the numbers must be in the same units.

$$\begin{aligned}\text{Unit price} &= \frac{\text{cost}}{\text{volume}} \\ &= \frac{\$5.49}{1890 \text{ mL}} \\ &= \$0.00290/\text{mL}\end{aligned}$$

$$\text{C } 5.49 \div 1890 \equiv 0.0029048$$

The unit price is \$0.00290/mL or 0.290¢/mL.

The unit price for the 1.89-L container is less than the unit prices of the other two containers. The best buy is the 1.89-L container for \$5.49.

\$1 = 100¢
To convert dollars to cents, multiply by 100.

1 L = 1000 mL
1.89 L = 1000 × 1.89
= 1890 mL

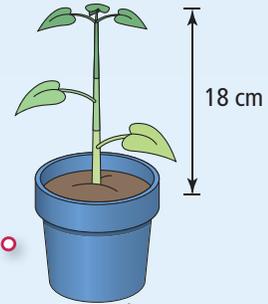
Show You Know

At Ed's Grocery, one brand of salsa is sold in the following container sizes. Which container of salsa is the best buy? Show your work.



Key Ideas

- A rate is a comparison of two quantities measured in different units.
 - A rate can be expressed as a fraction that includes the two different units. A rate cannot be expressed as a percent because a percent is a ratio that compares quantities expressed in the same units.



$$\text{Growth rate} = \frac{18 \text{ cm}}{3 \text{ months}}$$

The plant grew 18 cm in 3 months.

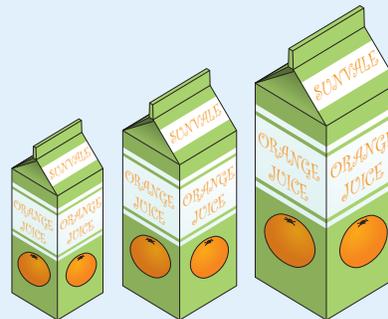
The growth rate compares height (in centimetres) and time (in months).

- A unit rate is a rate in which the second term is one.

$$\frac{18 \text{ cm}}{3 \text{ months}} = \frac{6 \text{ cm}}{1 \text{ month}}$$

The plant grew at a rate of 6 cm/month.

- A unit price is a unit rate that makes it easier to compare the cost of similar items.



0.408¢/mL 0.316¢/mL 0.290¢/mL
 $0.290\text{¢}/\text{mL} < 0.316\text{¢}/\text{mL} < 0.408\text{¢}/\text{mL}$
 The largest container is the best buy.

Communicate the Ideas

- Give an example of a ratio using words and numbers from the table.
 - What is a rate? Make up an example of a rate from the table.
 - Convert the rate in part b) to a unit rate.
- Two brands of canned dog food are on sale. Assume that the cans are the same size. Brand A costs \$13.60 for eight cans and Brand B costs \$8.75 for five cans. Explain how to find the unit price for Brands A and B. Explain how unit prices help you compare the cost of dog food.

Bear	Birth Mass (kg)	Mass After 60 Days (kg)
Black	0.3	6.5
Polar	0.7	7.4

3. a) Give two examples of rates that are common in every day life. Share your examples with a classmate.
 b) What units measure each of the rates in part a)?
 c) Explain why a rate cannot be expressed as a percent.

Check Your Understanding

Practise

For help with #4 to #6, refer to Example 1 on pages 56–57.

4. Determine the unit rate in each situation.
- An orca swims 110 km in 2 h.
 - A Canada goose flies 800 km in 12.5 h.
 - Cathy plants 45 daffodils in 30 min.
5. What is the unit rate in each?
- A blue whale eats 8 t of krill in 2 days.
 - The cruising speed of a blue whale allows it to travel 193 km in 10 h.
 - A bull moose bellows 15 times in $2\frac{1}{2}$ h.
6. Gina earns \$78.00 for working 6 h. Asad makes \$192.50 after working 14 h. Determine each person's unit rate of pay. Who has a greater hourly rate of pay?

For help with #7 to #9, refer to Example 2 on pages 57–58.

7. The table shows the price of different-sized packages of mixed nuts.

Nut Package	Mass	Price
1	300 g	\$2.19
2	500 g	\$3.09
3	700 g	\$4.83

- a) What is the unit price per 100 g for each package?

- b) Which package is the best buy? Explain your choice.

8. Fraser is shopping for milk. It is available in three sizes.



- What is the unit price for each carton of milk?
 - What is the unit price per 100 mL for the 1-L carton?
 - Which carton of milk is the best buy? Explain why.
9. Mala is shopping for honey. Her favourite brand is available in two sizes.



- Estimate which is the better buy. Show your thinking.
- Determine the better buy. Show your work.

Apply

10. Trevor rode his mountain bike 84 km in 3 h. Jillian rode 70 km in 2.5 h. Who is the faster cyclist? How do you know?
11. Shannon buys 12 granola bars for \$9.96.
- Determine the price per bar. Give your answer in dollars and cents.
 - Explain whether your answer in part a) is a ratio or a rate.
12. The rate at which glaciers melt is increasing globally. The Saskatchewan Glacier near Banff has receded 1.5 km in the last 75 years. The Peyto Glacier shown below receded 1320 m from 1923 to 1993. Which glacier had the greater annual rate of melting?



13. The table shows driving information for three drivers. Metric fuel consumption is measured in L/100 km, or litres per kilometre.

Driver	Distance (km)	Fuel Used (L)
Joe	400	28
Sarah	840	60
Martin	245	20

- What is the fuel consumption for Sarah's vehicle in litres per kilometre? Give your answer to four decimal places.
- How could you change the answer in part a) to express it in L/100 km?
- Which driver's vehicle had the lowest fuel consumption?

14. Conversion rates among currencies vary from day to day. The numbers in the table give the value in foreign currency of one Canadian dollar on one particular day.

Canadian	U.S.	Australian	European Union
1.00 dollar	0.8857 dollars	1.1527 dollars	0.6940 euros

- What was the value of \$600 Canadian in euros?
 - What was the value of \$375 Canadian in U.S. dollars?
 - What was the value of \$450 Canadian in Australian dollars?
15. Cindy Klassen from Winnipeg, Manitoba, won five speed skating medals at the 2006 Olympics. As of March 2006, she held the world record in the 1000 m, the 1500 m, and the 3000 m distances. Her times are shown in the table.

Time (min : s)	Distance (m)
1 : 13.11	1000
1 : 51.79	1500
3 : 53.34	3000



- Express each time in seconds.
- What was Cindy's speed in metres per second for her 1500 m record?
- How far does she skate in 10 s for the 3000 m distance?

Extend

16. Twins, Daniel and Grace, take turns mowing the lawn. Last week Grace mowed the lawn in 45 min. This week Daniel mowed the lawn in 40 min.
- What is the average mowing rate per hour for each twin? Give each answer to the nearest hundredth.
 - What is the difference between Daniel's and Grace's mowing rates?
17. The time it takes a planet to make one revolution of its axis is a day on that planet. Consider each planet to be a sphere. So, if you are standing on the equator of a planet, you are travelling in a circle as the planet spins on its axis. Use the table to find the rotation rate in kilometres per hour for each planet.

Planet	Radius at Equator (km)	Length of Day (h)
Venus	6 051	2 808
Earth	6 378	24
Saturn	60 268	10 233

The formula relating the circumference, C , of a circle to its radius is $C = 2 \times \pi \times r$.

18. Chad went to the bank to get some U.S. dollars for a trip to the Grand Canyon. He paid \$500 Canadian and received \$441.15 U.S.
- What was the conversion rate for exchanging Canadian dollars to U.S. dollars? Give your answer to four decimal places. What does your answer represent?
 - How many U.S. dollars would Chad receive for \$700 Canadian at the rate in part a)?
 - Two days later, Chad returned to the bank and converted the \$441.15 U.S. back to Canadian dollars. He received only \$492.25 Canadian. What was the bank's conversion rate on that day for exchanging U.S. dollars to Canadian dollars? Give your answer to four decimal places.
 - How many U.S. dollars would Chad receive for \$700 Canadian at the rate in part c)?
19. Express 60 km/h in metres per second.

MATH LINK

Kheer is a traditional rice pudding made in India and Pakistan. Pakistani kheer tends to be thicker than the Indian version. Look at the recipe for kheer. If the original recipe serves four people, calculate the quantity of each ingredient you need to serve 10 people. Use ratios and rates to support your reasoning.



Kheer

Ingredients:

- 125 mL rice (basmati)
- 1 L milk
- 50 mL raisins
- 250 mL sugar
- 5 mL cardamom (or nutmeg)
- 50 mL almonds (slivered)

Method:

- Wash rice well.
- Boil milk and add rice. Simmer on low heat until rice is soft, stirring frequently to prevent sticking.
- When the rice is cooked and the mixture gets a semi-thick creamy consistency, add sugar and stir well.
- Remove from heat and add cardamom, slivered almonds, and raisins.
- Serve warm or chilled.

2.3

Proportional Reasoning

Focus on...

After this lesson, you will be able to...

- solve problems using proportional reasoning
- use more than one method to solve proportional reasoning problems

When you go snowboarding or skiing, you use proportional reasoning to determine the correct length of ski or board for you to use. This involves using the ratio of your height to the length of the ski or board. To determine the correct width of the board so that your feet do not hang over, you use the ratio of the waist of the board, which is the narrowest part, to your boot size. Riders with small feet need narrower boards than riders with big feet.

When you draw a portrait in art class, you use proportional reasoning to figure out how large to make each facial feature in relation to the other features and how to align the features on the head.

Think about where you have made comparisons. Where might you have used proportional reasoning?



Explore the Math

How do you use proportional reasoning?

1. Copy the following table into your notebook. Put two rows under the column headings to record data for two typists.

Student	Number of Words	Time (min)
		4
		4

2. Work with a partner. Select a short story to type.

Materials

- computer access
- short story
- stopwatch
- sticky notes

Tech Link

You can use a word processing program to count the words for you.

Did You Know?

Typing performance involves both speed and accuracy. Many computer programs provide information about words per minute, number of keystrokes, and error rates.

3. Use a stopwatch and take turns to time each other's typing. The timer tells the typist when to begin and when to stop (after 4 min).
4.
 - a) Type at a comfortable rate so you can avoid making many errors.
 - b) After the time is up, mark your stopping place in the text using a sticky note.
 - c) Count and record the total number of words typed in 4 min.
5. Trade roles and repeat step 4 to get data for your partner.
6.
 - a) What is the four-minute typing rate for each typist?
 - b) What is the unit rate for each typist?

Reflect on Your Findings

7.
 - a) If each typist continued typing at the same rate, how many words could each person type in 1 h? Approximately how many pages is that?
 - b) What other factors might affect how long it takes to type the entire story? Give an estimate of the time needed for each typist to type the story.
 - c) How did you use a **proportion** to find your answer to part a)?

proportion

- a relationship that says that two ratios or two rates are equal
- can be written in fraction form:

$$\begin{array}{c} \times 3 \\ \frac{2}{3} = \frac{6}{9} \\ \times 3 \\ \times 3 \\ \frac{2 \text{ km}}{3 \text{ h}} = \frac{6 \text{ km}}{9 \text{ h}} \\ \times 3 \end{array}$$

What number do you divide the denominator by? What number do you divide the numerator by? Are both numbers the same? Why?

$$\frac{11.58\text{¢}}{2 \text{ kWh}} = \frac{5.79\text{¢}}{1 \text{ kWh}}$$

Electricity costs 5.79¢ per kWh or 5.79¢/kWh.

$$30 \text{ kWh costs } 30 \times 5.79\text{¢} = 173.7\text{¢}$$

So, 30 kWh costs 174¢ or \$1.74 rounded to the nearest cent.

173.7 means 173 and seven tenths cents. Recall that five tenths or more is rounded to the next cent.

$$\boxed{C} \quad \boxed{30} \times \boxed{5.79} = \boxed{173.7}$$

Example 1: Solve a Rate Problem Using Proportional Reasoning

Electricity costs 11.58¢ for 2 kWh. How much does 30 kWh cost? Give your answer to the nearest cent.

Solution

Method 1: Use a Unit Rate

A cost of 11.58¢ for 2 kWh can be expressed as the rate $\frac{11.58\text{¢}}{2 \text{ kWh}}$.

Determine the unit rate.

Method 2: Use a Proportion

Make a proportion to show what you want to find.

$$\frac{11.58\text{¢}}{2 \text{ kWh}} = \frac{\blacksquare}{30 \text{ kWh}}$$

Solve the proportion.

$$\frac{11.58\text{¢}}{2 \text{ kWh}} = \frac{\blacksquare}{30 \text{ kWh}}$$

$$11.58\text{¢} \times 15 = 173.7\text{¢} \quad \boxed{C} \quad \boxed{11.58} \times \boxed{15} = \boxed{173.7}$$

So, 30 kWh costs about 174¢ or \$1.74.

What number do you multiply the denominator by? What number do you multiply the numerator by?

11.58¢ is about 12¢.
 $12 \times 15 = (10 \times 15 + 2 \times 15)$
 $= 150 + 30$
 $= 180$
The answer will be about 180¢ or \$1.80.



Show You Know

There are 72 players on 8 baseball teams. Determine the number of players on 2 teams. Show how to find the answer more than one way.

Example 2: Solve a Ratio Problem Using Proportional Reasoning

A wildlife biologist wants to know how many trout are in a slough in Saskatchewan. He captures and tags 24 trout and releases them back into the slough. Two weeks later he returns and captures 30 trout and finds that 5 of them are tagged. He uses the following ratios to estimate the number of fish in the slough:

$$\frac{\text{fish recaptured with tags}}{\text{total fish recaptured}} = \frac{\text{fish caught and tagged}}{\text{total fish in slough}}$$

How many trout does he estimate are in the slough?

Solution

Method 1: Use a Proportion in Lowest Terms

$$\frac{\text{fish recaptured with tags}}{\text{total fish recaptured}} = \frac{\text{fish tagged}}{\text{total fish in slough}}$$

$$\frac{5}{30} = \frac{24}{t} \quad \text{Set up the proportion using equal ratios.}$$

$$\frac{1}{6} = \frac{24}{t} \quad \text{Reduce } \frac{5}{30} \text{ to } \frac{1}{6}.$$

$$\times 24$$

$$\frac{1}{6} = \frac{24}{t} \quad \text{Make equivalent ratios.}$$

$$\times 24$$

$$t = 6 \times 24 = 144$$

The biologist estimates there are 144 trout in the slough.

Literacy Link

In Western Canada, a *slough* is a small lake or pond formed by rain or melted snow.

Did You Know?

Wildlife biologists can show that these ratios are equal if the fish population has an opportunity to mix before the recapture.

Strategies

Use a Variable

Method 2: Use the Original Proportion

How would you find the solution if you did not write $\frac{5}{30}$ in lowest terms?

$$\frac{5}{30} = \frac{24}{t}$$

$$\times 4.8$$

$$\frac{5}{30} = \frac{24}{t}$$

$$\times 4.8$$

$$t = 30 \times 4.8$$

$$t = 144$$

Divide to find what number multiplied by 5 gives 24.

$$\frac{24}{5} = 4.8$$

Make equivalent ratios.

The biologist estimates there are 144 trout in the slough.

Show You Know

How much will a dozen erasers cost if three erasers cost 75¢? Show how to find the answer in more than one way.

Key Ideas

- A proportion is a relationship that says that two ratios or two rates are equal.
 - A proportion can be expressed in fraction form:

$$\frac{1}{2} = \frac{5}{10}$$

$\times 5$ (top arrow), $\times 5$ (bottom arrow)

$$\frac{60 \text{ sit-ups}}{3 \text{ min}} = \frac{20 \text{ sit-ups}}{1 \text{ min}}$$

$\div 3$ (top arrow), $\div 3$ (bottom arrow)

- You can solve proportional reasoning problems using several different methods. A potato farmer can plant three potato plants per 0.5 m^2 . How many potato plants can she plant in an area of 85 m^2 ?

- Use a unit rate. $\frac{3 \text{ plants}}{0.5 \text{ m}^2} = \frac{6 \text{ plants}}{1 \text{ m}^2}$ The unit rate is 6 plants/ m^2 .

$$6 \times 85 = 510$$

The farmer can plant 510 potato plants.

- Use a proportion.

$$\frac{3 \text{ plants}}{0.5 \text{ m}^2} = \frac{\blacksquare}{85 \text{ m}^2}$$

$\times 170$ (top arrow), $\times 170$ (bottom arrow)

Missing value is $3 \times 170 = 510$

The farmer can plant 510 potato plants.

Communicate the Ideas

1. Explain the similarities and differences between a ratio, a rate, and a proportion. Give an example of each one.
2. Your friend missed the lesson on proportions. Explain how to use a proportion to solve this problem.
Cheryl is selling marbles. What is the cost of seven marbles?

Marbles	2	3	4
Cost (¢)	70	105	140

3. a) Write a proportion based on the following scenario:
Three balls cost \$1.25. What is the cost of 12 balls?
b) Solve the proportion.

Check Your Understanding

Practise

For help with #4 to #9, refer to Example 1 on pages 64–65.

4. Determine the unit rate.
 - a) Three dinner rolls cost 99¢.
 - b) Seven identical objects have a mass of 14 kg.
5. What is the unit rate in each?
 - a) Two pens cost 94¢.
 - b) Four blocks stacked one on top of the other are 24 cm high.
6. Delia was paid \$35 for 5 h of babysitting. How much should she receive for 3 h? Use a unit rate to find the answer.
7. Solve #6 using a proportion. Show how to find the answer more than one way.
8. Determine the missing value.

a) $\frac{2}{3} = \frac{\blacksquare}{15}$	b) $\frac{\blacksquare}{5} = \frac{14}{35}$
c) $\frac{30}{45} = \frac{6}{\blacksquare}$	d) $\frac{3}{\blacksquare} = \frac{12}{36}$

9. Determine the missing value to make each rate equivalent. Include the units.

a) $\frac{60 \text{ km}}{3 \text{ h}} = \frac{\blacksquare}{6 \text{ h}}$	b) $\frac{\$3}{4 \text{ cans}} = \frac{\$15}{\blacksquare}$
c) $\frac{178 \text{ beats}}{2 \text{ min}} = \frac{\blacksquare}{1 \text{ min}}$	d) $\frac{48 \text{ km}}{\$16} = \frac{192 \text{ km}}{\blacksquare}$

For help with #10 to #14, refer to Example 2 on pages 65–66.

10. Set up a proportion for each situation.
 - a) If 10 beans have a mass of 17 g, then 30 beans have a mass of 51 g.
 - b) There are 13 boys for 15 girls in every classroom at Albany Middle School. If there are 65 boys in the school, then there are 75 girls.
 - c) On a map, 1 cm represents 25 km. Kendra wants to ride her bike 160 km. This distance is 6.4 cm on the map.
11. A small gear turns 18 times in the same time that a large gear turns 4 times. How many times will the large gear turn if the small gear turns 54 times? Draw a diagram to help set up a proportion and solve the problem.

- 12.** Set up a proportion for each situation using a variable. Do not find the answer.
- Walter makes his own oil and vinegar dressing. His recipe calls for 175 mL of olive oil and 50 mL of vinegar. What amount of vinegar does he need to mix with 300 mL of olive oil?
 - A baseball player has a home run to strikeouts ratio of 3 : 17. How many home runs should he hit if he strikes out 187 times?
- 13.** Two quarters have the same value as ten nickels. What is the value of five quarters in nickels?
- 14.** Last night 30 cm of snow fell in 6 h. If it continues snowing at the same rate, how long will it take for 45 cm of snow to fall? Determine the answer two different ways.

Apply

- 15.** Look at the pattern. Set up a proportion you could use to find the number of small squares in Figure 7.



Figure 1



Figure 2

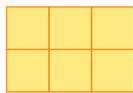


Figure 3

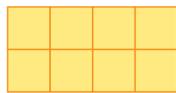


Figure 4

- 16.** A gardener takes a half hour to mow and weed a lawn that measures 20 m by 15 m. He charges \$25 per hour. How much should the gardener receive for a lawn that measures 40 m by 30 m?

- 17.** Fresh pickerel is advertised in a local market.



1 kg = 1000 g

- How much will 6 kg of pickerel cost?
- Use a proportion to find the cost of 1600 g of pickerel.

- 18.** At an amusement park, a new thrill ride was introduced. It costs \$7.50 for 3 rides on the Wild Slider.
- What is the Wild Slider's unit rate per ride?
 - At this rate, what would it cost for 18 rides on Wild Slider? Determine the answer two different ways.

- 19.** Determine the missing value in each equivalent fraction.

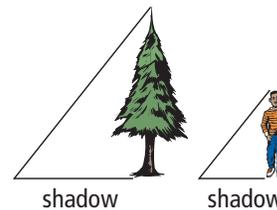
a) $\frac{3}{\square} = \frac{18}{24} = \frac{\square}{12}$

b) $\frac{48 \text{ km}}{\$16} = \frac{144 \text{ km}}{\square} = \frac{\square}{\$64}$

- 20.** A breakfast cereal contains corn, wheat, and rice in the ratio of 3 to 4 to 2. If a box of cereal contains 225 g of corn, how much rice does it contain?
- 21.** David can saw a log into three pieces in 7 min. If he continues sawing at a constant rate, how long will it take him to saw a similar log into six pieces?

- 22.** The height of an object compared to the length of its shadow is constant for all objects at any given time.

$$\frac{\text{tree height}}{\text{length of shadow}} = \frac{\text{student height}}{\text{length of shadow}}$$



Use this information to help answer the following questions.

- If a 15-m tree casts a 9-m shadow, what is the height of a student who casts a 1.08-m shadow?
- If a 50-m tower has a shadow 16 m long, how long is the shadow of a student who is 1.5 m tall? Give your answer to the nearest centimetre.

23. According to the *Guinness Book of World Records*, the world's smallest horse is Thumbelina. Thumbelina is 42.5 cm tall and eats about 0.3 kg of food per day. A former world record holder ate food in the same proportion to its height. If it was 46.25 cm tall, how much did it eat? Give your answer to the nearest hundredth of a kilogram.

24. a) Describe a pattern you could use to find the next fraction in the following set of fractions. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$

b) Describe a pattern you could use to find the next fraction in the following set of fractions. $\frac{5}{6}, \frac{10}{12}, \frac{15}{18}$

c) Choose any pair of fractions from part a) or part b). Multiply the numerator of one fraction by the denominator of the other fraction. Repeat for two other fractions in the same set. What do you notice about the two products?

d) What prediction could you make about the cross-products of any pair of equivalent fractions? Test your prediction on another pair of equivalent fractions.

Extend

25. Mark estimates that frogs eat six insects per hour and that dragonflies eat nine insects per hour. Assume a frog rests for 8 h each day and a dragonfly rests for 13 h each day. Neither eats while resting.

a) Determine the daily rate of insects eaten by a frog and a dragonfly. Which one eats more insects per day?

b) How many insects would a dragonfly eat in a week?

c) How many insects would a frog eat in August?

26. Two circles have radii with a ratio of 1 to 2. Use a diagram to help answer the following questions.

a) What is the ratio of their circumferences?

b) What is the ratio of their areas?

27. If $a:b = 4:5$, find the ratio of $5a:7b$.

28. The dosage of a certain medicine for a child is 2.5 mL for each 3 kg mass of the child. What is the dose, in millilitres, for a child with a mass of 16.5 kg?

MATH LINK

A horiatiki Greek salad has tomatoes, cucumbers, feta cheese, and olives. It does not contain any lettuce. Many cultures have similar salads. For example, ezme salatasi is a Turkish tomato-cucumber salad with red peppers and paprika, but without the feta and olives.

a) It costs \$7.60 to make the horiatiki salad for 12 people. What is the unit price?

b) Choose and write down a recipe for a soup, a salad, or an appetizer that serves between 4 and 6 people. Record how much of each ingredient you will need to serve 10 people at your international meal.



Horiatiki Salata

Yield: 4 Servings

Ingredients:

- 3 to 4 tomatoes
- 1 cucumber
- 1 red onion
- 125 mL Kalamata olives
- 125 g feta cheese
- 250 mL dressing (olive oil, red wine vinegar, garlic, oregano, salt and pepper)