

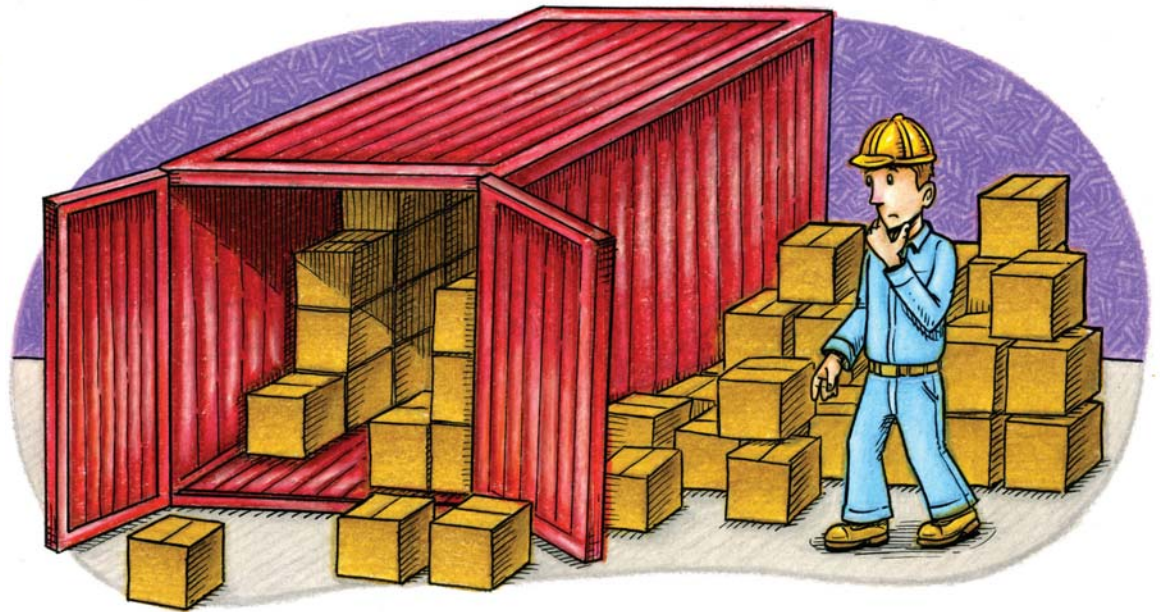
7.1

Understanding Volume

Focus on...

After this lesson, you will be able to...

- explain the meaning of volume
- determine the volume of a right rectangular prism, right triangular prism, and right cylinder
- show that orientation does not affect volume



Bruce has just taken on a part-time job at a local shipping company. He is packing boxes into a shipping container. He knows how many boxes he can fit on the bottom of the container. How can he use this information to figure out how many boxes the shipping container will hold?

Materials

- centimetre cubes

base (of a prism or cylinder)

- any face of a prism that shows the named shape of the prism
- the base of a rectangular prism is any face
- the base of a triangular prism is a triangular face.
- the base of a cylinder is a circular face

height (of a prism or cylinder)

- the perpendicular distance between the two bases of a prism or cylinder

Explore the Math

How does the area of the base of a right prism relate to its volume?

1. **a)** Use centimetre cubes to build models of four different right rectangular prisms.
- b)** What is the area of the **base** for each model? Record your data.
- c)** What is the **height** of each model? Record your data.

2. How does the number of cubes help to determine the **volume** of each rectangular prism? What is the volume of each prism? Record your data.

One centimetre cube is equal to 1 cm^3 .

volume

- the amount of space an object occupies
- measured in cubic units

Reflect on Your Findings

4. a) What is the relationship between the area of the base, the height of the prism, and the volume of a rectangular prism?
- b) Do you think this same relationship exists for the volume of a right triangular prism? Explain your reasoning.

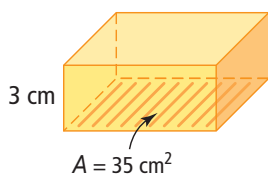
Literacy Link

Read 1 cm^3 as "one cubic centimetre."

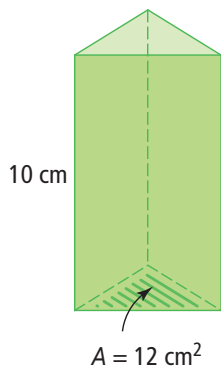
Example 1: Determine the Volume Using the Base and the Height

Determine the volume of each right prism or cylinder.

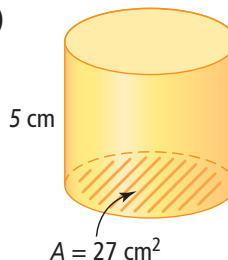
a)



b)



c)



Literacy Link

Prisms and cylinders in this chapter are *right* prisms and *right* cylinders.

Solution

- a) The prism is a right rectangular prism.
The area of the rectangular base is 35 cm^2 .
The height of the prism is 3 cm.

Volume = area of base \times height of prism

$$V = 35 \times 3$$

$$V = 105$$

The volume of the right rectangular prism is 105 cm^3 .

Why are the units for volume in cm^3 ?

- b) The prism is a right triangular prism.
The area of the triangular base is 12 cm^2 .
The height of the prism is 10 cm.

Volume = area of base \times height of prism

$$V = 12 \times 10$$

$$V = 120$$

The volume of the right triangular prism is 120 cm^3 .

- c) The cylinder is a right cylinder.
 The area of the circular base is 27 cm^2 .
 The height of the cylinder is 5 cm .
 Volume = area of base \times height of cylinder
 $V = 27 \times 5$
 $V = 135$
 The volume of the right cylinder is 135 cm^3 .

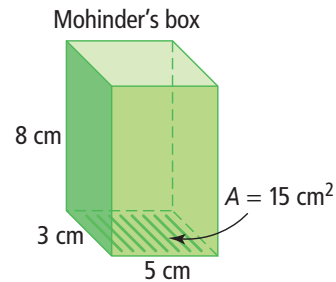
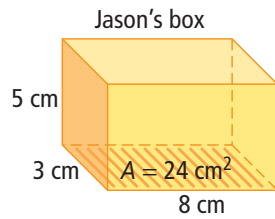
Show You Know

What is the volume of the right cylinder?



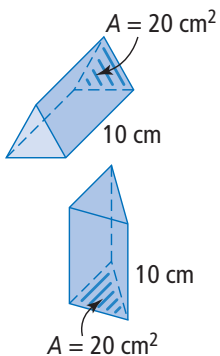
Example 2: Determine the Volume Using Different Orientations

Jason and Mohinder have two boxes with the same dimensions, $5 \text{ cm} \times 3 \text{ cm} \times 8 \text{ cm}$. Jason's box is short, with a height of 5 cm . Mohinder's box is taller; its height is 8 cm . Mohinder says his box has a larger volume than Jason's box. Is he correct?



orientation

- the different position of an object formed by translating, rotating, or reflecting the object



Solution

Determine the volume of each rectangular prism.

Jason's box: Base area of 24 cm^2

Volume = area of base \times height

$$V = 24 \times 5$$

$$V = 120$$

The volume of the rectangular prism is 120 cm^3 .

Mohinder's box: Base area of 15 cm^2

Volume = area of base \times height

$$V = 15 \times 8$$

$$V = 120$$

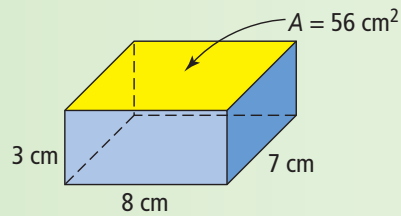
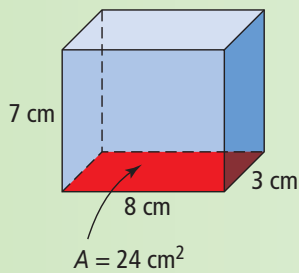
The volume of the rectangular prism is 120 cm^3 .

Mohinder is not correct. Both boxes have the same volume.

Do you think changing the **orientation** of a 3-D object ever affects the volume?

Show You Know

Which box has the greater volume? Explain your reasoning.



Key Ideas

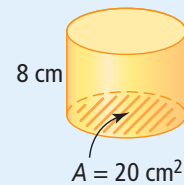
- The volume of a right cylinder or a right prism can be determined by multiplying the area of the base by the height of the cylinder or prism.

Volume = area of base \times height of cylinder

$$V = 20 \times 8$$

$$V = 160$$

The volume of the cylinder is 160 cm^3 .

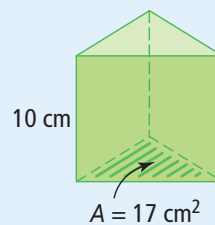


Volume = area of base \times height of prism

$$V = 17 \times 10$$

$$V = 170$$

The volume of the triangular prism is 170 cm^3 .



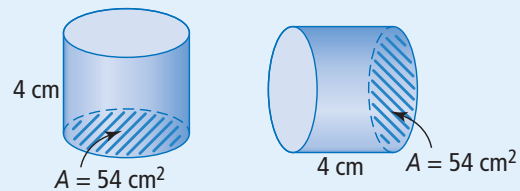
- Changing the orientation of a 3-D object does not affect its volume.

Volume = area of base \times height

$$V = 54 \times 4$$

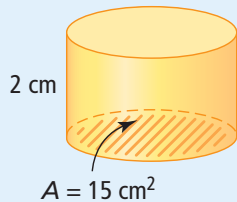
$$V = 216$$

The volume of the cylinder is 216 cm^3 .



Communicate the Ideas

1. Evan calculated the volume of a right cylinder. Charlotte calculated the volume of a right rectangular prism. Did either of them make an error in their solutions? Explain how you know.

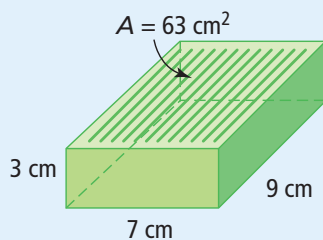


Volume = area of base \times height

$$V = 15 \times 2$$

$$V = 30$$

The volume of the cylinder is 30 cm^3 .



Volume = area of base \times height

$$V = 63 \times 7$$

$$V = 441$$

The volume of the rectangular prism is 441 cm^3 .



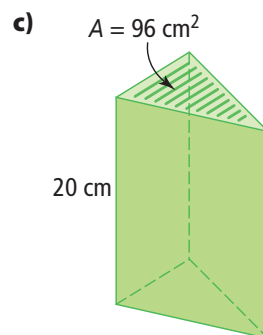
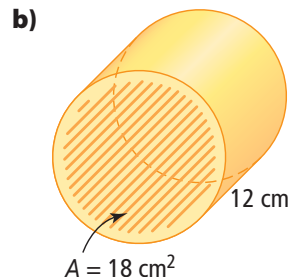
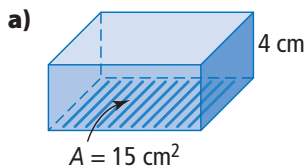
2. Does the volume of a right prism depend on which face is used as the base in the calculations? Use examples to support your position.

Check Your Understanding

Practise

For help with #3 and #4, refer to Example 1 on pages 247–248.

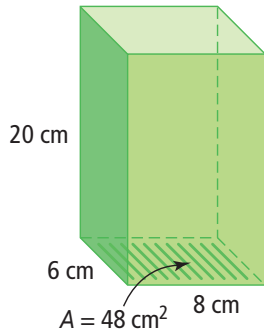
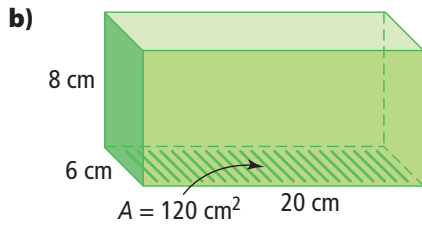
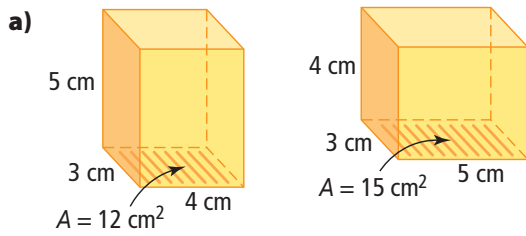
3. Determine the volume of each right prism or cylinder.



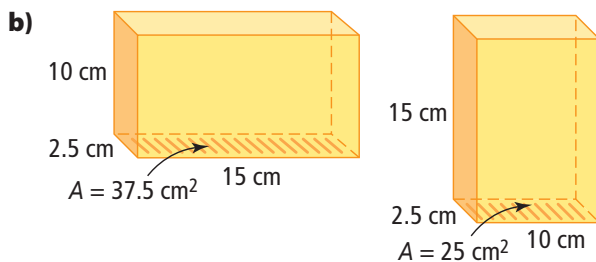
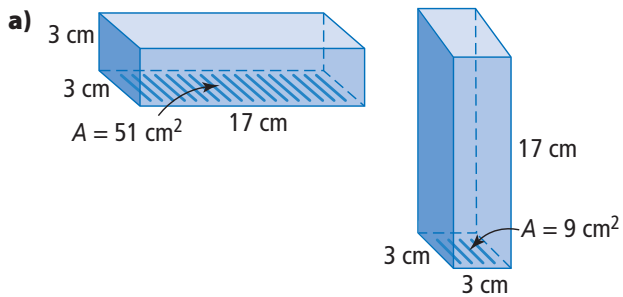
4. What is the volume of each right prism?
- area of base = 12 cm^2 , height = 8 cm
 - area of base = 18 cm^2 , height = 4 cm
 - height = 9 cm, area of base = 14 cm^2

For help with #5 and #6, refer to Example 2 on page 248.

5. Determine the volume of each right rectangular prism.



6. What is the volume of each right rectangular prism?



Apply

7. What is the height of each of the following right rectangular prisms?

a) volume = 32 cm^3 , area of base = 8 cm^2

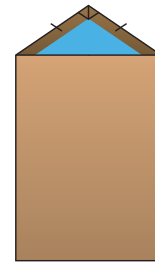
b) volume = 35 cm^3 , area of base = 5 cm^2

c) area of base = 9 cm^2 , volume = 36 cm^3

8. Nina uses 15 centimetre cubes to make the base of a rectangular prism. Determine the volume if the prism has a total of 5 layers of cubes. Show your thinking.

9. How many ways can you build a rectangular prism from 16 centimetre cubes? Use diagrams or centimetre cubes to show your designs.

10. A water trough is in the shape of a right triangular prism with base area of 1250 cm^2 and a height of 100 cm. How much water can be put in before it overflows?



11. José is having vegetable soup. The area of the base of the soup can is 10.4 cm^2 , and the height is 10 cm. When José opens the can, he sees that the soup comes up to a height of only 9 cm. What volume of soup is in the can?



12. Bill is building a wooden sandbox with a base area of 8 m^2 for his granddaughters. He does not want to order more than 1.5 m^3 of sand to fill it. He has enough wood to build the sandbox up to 0.22 m deep. What is the minimum height he should build the sandbox to allow the sand to be spread evenly? Justify your answer.

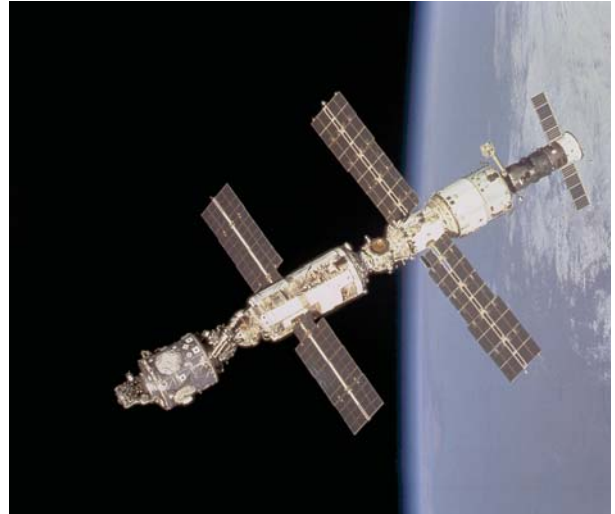


13. Ocean City Aquarium is building a new tank for its coral reef fish. The area of the base is $18\,750 \text{ cm}^2$ and the height is 90 cm .
- What is the volume of the tank in cubic centimetres?
 - What is the volume in litres?

$1 \text{ L} = 1000 \text{ cm}^3$



14. One of the solar arrays on the International Space Station is a rectangular prism with a base area of 892 m^2 and a thickness of 27.5 m . What is the volume of one solar array?



Literacy Link

The word *thick* is sometimes used to describe the height of an object.

2 cm thick



15. The International Space Station is shaped like a cylinder that has a cross-sectional area of 615 m^2 and a length of 44.5 m . The living space for the astronauts is 425 m^3 . What percent of the volume of the space station is used for living?

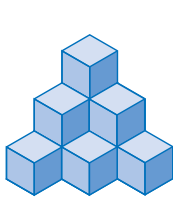
The cross-sectional area is the area of the circle you see if you cut across the cylinder.

Web Link

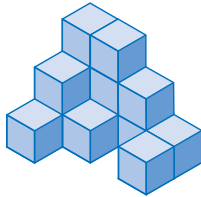
To learn more about the International Space Station, go to www.mathlinks8.ca and follow the links.

Extend

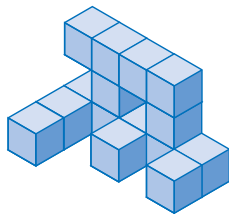
16. In the structures below, each small cube has a base area of 4 cm^2 and a height of 2 cm. In the first two structures, assume the side facing away from you is solid.



Structure 1



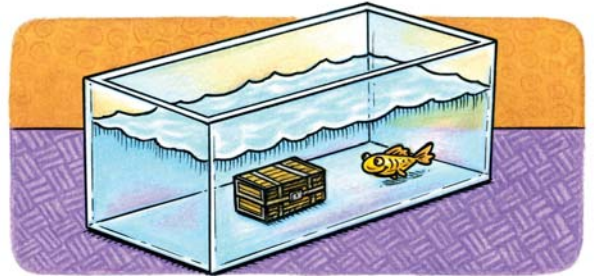
Structure 2



Structure 3

- How many cubes are in each structure?
 - What is the least number of small cubes needed to complete each structure so that it becomes a rectangular prism?
 - What is the total number of cubes in each completed structure?
 - What is the volume of each completed rectangular prism?
- What is the ratio of the volume of the cube to the volume of the box?
 - What is the ratio of the area of the base of the cube to the area of the base of the box?
 - What is the ratio of the height of the cube to the height of the box?
 - What relationship exists among these three ratios?

17. Callie's rectangular fish tank has a base area of 800 cm^2 and contains water to a depth of 15 cm. She adds a solid decoration in the shape of a rectangular prism to the bottom of the tank. The decoration has a base area of 40 cm^2 and a height of 5 cm. What is the new level of water in the tank?

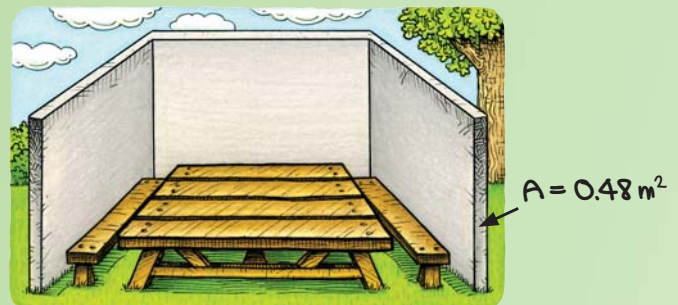


18. A cube with a base area of 4 cm^2 and a height of 2 cm is inside a box with a base area of 16 cm^2 and a height of 4 cm.
- What is the ratio of the volume of the cube to the volume of the box?
 - What is the ratio of the area of the base of the cube to the area of the base of the box?
 - What is the ratio of the height of the cube to the height of the box?
 - What relationship exists among these three ratios?

MATH LINK

Some parks have shelters around the eating areas. These shelters consist of two or three walls. The area of the end of each wall is 0.48 m^2 .

- Sketch and label the dimensions of a sheltered eating area. Keep in mind that the picnic table that will go inside is about 1.8 m long and 0.74 m wide.
- Calculate the volume of concrete used to make the walls.



7.2

Volume of a Prism

Focus on...

After this lesson, you will be able to...

- use a formula to determine the volume of a right rectangular prism
- use a formula to determine the volume of a right triangular prism

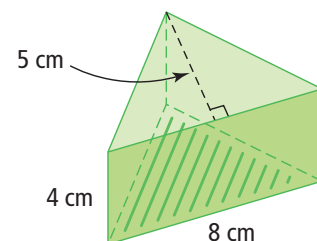
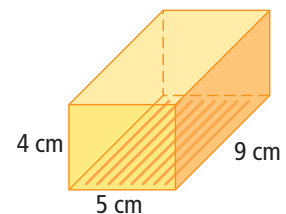


Recycling bins are located in most schools. Students and staff members fill them with paper, aluminum cans, plastic and glass bottles, and other recyclables. Schools probably recycle more paper than anything else. What would you need to know about a bin for recycled paper in order to determine how much paper was recycled?

Explore the Math

How can you use the dimensions to calculate the volumes of right prisms?

1. How is volume related to the area of the base and the height of a prism?
2. Show how would you could determine the volume of the right rectangular prism shown?
3. Show how you could determine the volume of the right triangular prism shown?

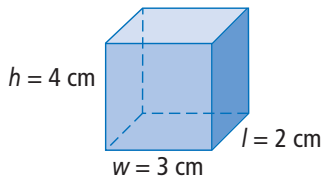


Reflect on Your Findings

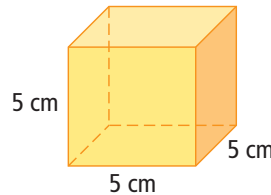
- If you know only the dimensions of a right rectangular prism, how can you determine the volume? Test your method using an example.
- If you know only the dimensions of the triangular base and the height of the prism, how can you determine the volume of a right triangular prism? Test your method using an example.
- How are the formulas for the volume of a right rectangular prism and a right triangular prism different? How are they the same?

Example 1: Use a Formula to Determine the Volume of a Right Rectangular Prism

- a) Determine the volume of the right rectangular prism.



- b) Determine the volume of the cube.



How do you calculate the area of a rectangle?

Solution

- a) Volume of a right rectangular prism = area of rectangular base \times height of prism

Volume of a right rectangular prism = (length \times width) \times height

$$V = l \times w \times h$$

$$V = 2 \times 3 \times 4$$

$$V = 24$$

The volume of the right rectangular prism is 24 cm^3 .

Strategies

Use a Variable

- b) A cube is also a right rectangular prism.

Volume of a cube = area of square base \times height of prism

Volume of a cube = (length \times width) \times height

$$V = s \times s \times s$$

$$V = 5 \times 5 \times 5$$

$$V = 125$$

The volume of the cube is 125 cm^3 .

Since $s \times s \times s = s^3$, you could write the formula as:

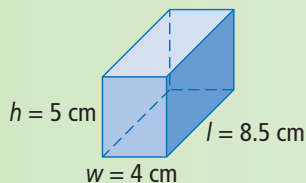
$$V = s^3$$

$$V = 5^3$$

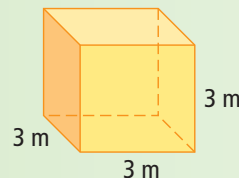
$$V = 125$$

Show You Know

- a) What is the volume of the right rectangular prism?



- b) What is the volume of a cube with edge length 3 m?



Example 2: Use a Formula to Determine the Volume of a Right Triangular Prism

Determine the volume of the right triangular prism.

Solution

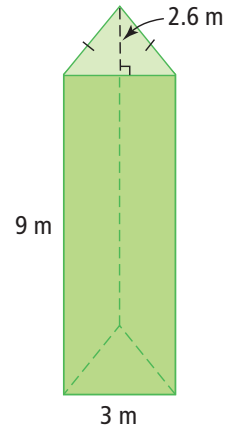
Volume of a triangular prism = area of triangular base
× height of prism

Volume of a triangular prism = (base × height ÷ 2)
× height of prism

$$V = (3 \times 2.6 \div 2) \times 9$$

$$V = 35.1$$

The volume of the right triangular prism is 35.1 m^3 .



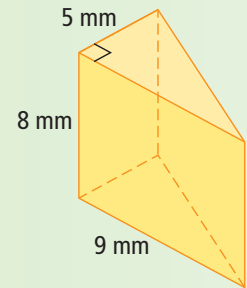
Strategies

Use a Variable

How do you calculate the area of a triangle?

Show You Know

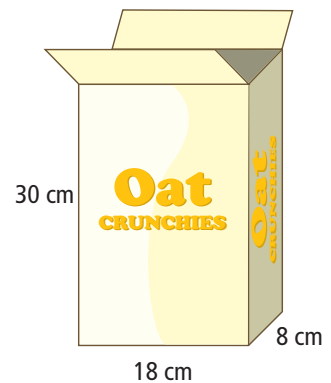
What is the volume of the right triangular prism?



Example 3: Use Volume to Solve a Problem

When Katie opened a new box of Oat Crunchies, she noticed that the box was only $\frac{5}{6}$ full.

- How much space was empty?
- Why do you think packages often seem to have empty space when first opened?



Solution

- The cereal box is a right rectangular prism.

$$V = l \times w \times h$$

$$V = 18 \times 8 \times 30$$

$$V = 4320$$

The volume of the cereal box is 4320 cm^3 .

The package is $\frac{5}{6}$ full. Therefore, $\frac{1}{6}$ of the package is empty.

$$\begin{aligned} \text{Amount of empty space} &= \frac{1}{6} \times 4320 \\ &= \frac{4320}{6} \\ &= 720 \end{aligned}$$

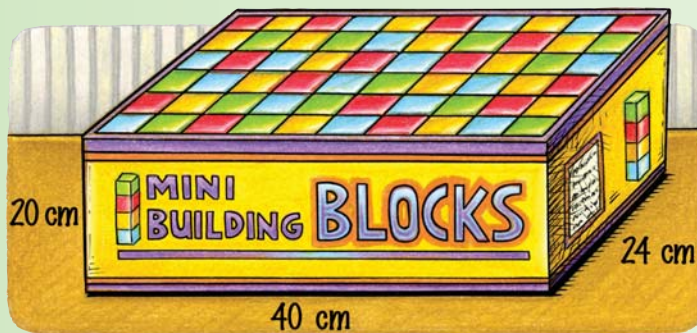
$$\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$$

There was 720 cm^3 of empty space.

- b) Packages often seem to have empty space because the contents settle when being shipped.

Show You Know

Mr. Chin bought a box of small building blocks for his four children. He will give an equal number to each of them. What volume of blocks will each child get?



Key Ideas

- The volume of a right rectangular prism can be determined using the formula:

$$V = l \times w \times h$$

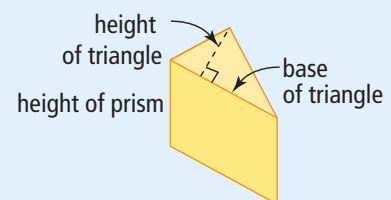
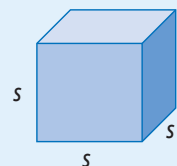
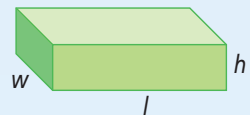
- The volume of a cube can be determined using the formula:

$$V = s \times s \times s$$

$$V = s^3$$

- The volume of a right triangular prism can be determined using the formula:

$$V = (\text{base of triangle} \times \text{height of triangle} \div 2) \times \text{height of prism}$$



Communicate the Ideas

- Grace tells Dakota that the volume of a cube can be found using the formula: $V = l \times w \times h$. Do you agree? Explain.
- Kwan wants to build a concrete ramp to his back door. He wants to determine the volume of concrete needed for the ramp. What measurements does he need to know? Justify your response.
- Jack's family opened a full carton of frozen yogurt for dessert. After they ate, there was $\frac{3}{4}$ left. Jack wants to know what volume of frozen yogurt they ate. He does the following calculation.

Volume of carton:

$$V = 12 \times 9 \times 18$$

$$V = 1944$$

The volume of the carton is 1944 cm^3 .

Volume of frozen yogurt eaten:

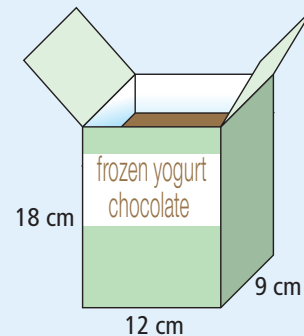
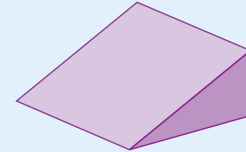
$$V = 1944 \times \frac{3}{4}$$

$$V = \frac{1944 \times 3}{4}$$

$$V = \frac{5832}{4}$$

$$V = 1458$$

They ate 1458 cm^3 of frozen yogurt.



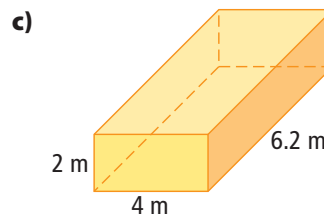
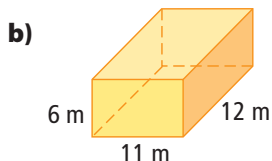
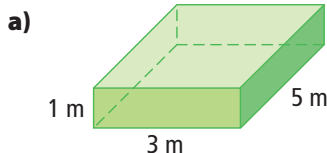
- What mistake did Jack make?
- Show the correct calculation.
- Show an alternative way to calculate the answer.

Check Your Understanding

Practise

For help with #4 to #6, refer to Example 1 on page 255.

- Determine the volume of each right rectangular prism.



- What is the volume of each right rectangular prism?

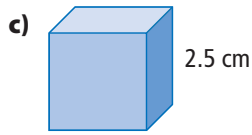
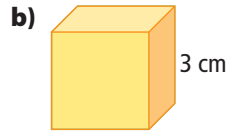
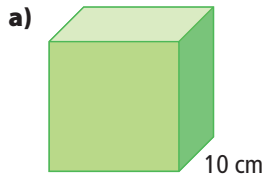
a) $l = 2 \text{ m}$, $w = 2 \text{ m}$, $h = 10 \text{ m}$

b) $l = 8 \text{ cm}$, $w = 7 \text{ cm}$, $h = 9 \text{ cm}$

c) $l = 11.7 \text{ mm}$, $w = 6.3 \text{ mm}$,
 $h = 2.9 \text{ mm}$

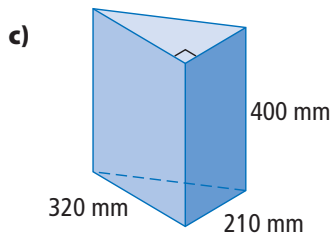
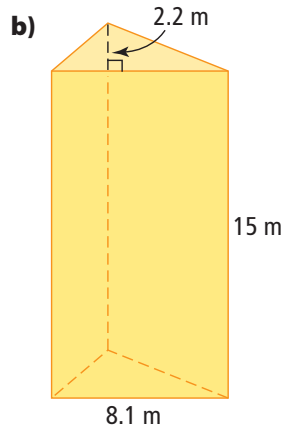
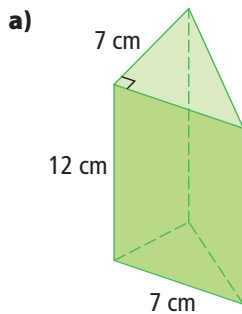
d) $l = 6.2 \text{ cm}$, $w = 6.4 \text{ cm}$, $h = 6.4 \text{ cm}$

6. Determine the volume of each cube.



For help with #7 and #8, refer to Example 2 on page 256.

7. Determine the volume of each right triangular prism.



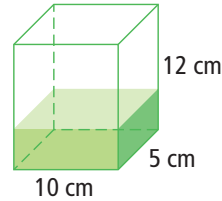
8. What is the volume of each right triangular prism?

- a) base of triangle = 3 m
height of triangle = 7 m
height of prism = 8 m
- b) base of triangle = 15 cm
height of triangle = 8 cm
height of prism = 20 cm
- c) base of triangle = 10 mm
height of triangle = 9.1 mm
height of prism = 11.3 mm

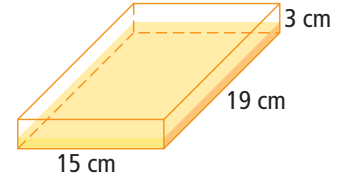
For help with #9 and #10, refer to Example 3 on pages 256–257.

9. Determine the volume of the contents of each right prism.

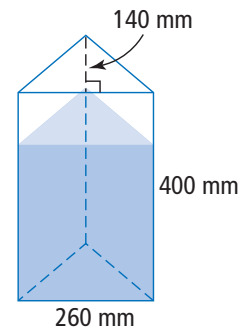
a) $\frac{1}{3}$ full



b) $\frac{3}{8}$ full

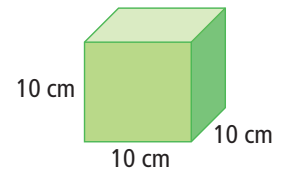


c) $\frac{3}{4}$ full

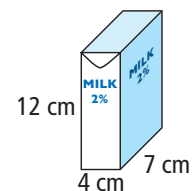


10. Determine the volume of the empty space in each object.

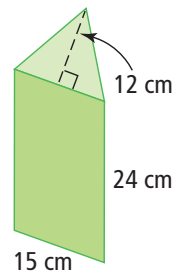
a) $\frac{4}{5}$ full of facial tissues



b) $\frac{3}{4}$ full of milk



c) $\frac{1}{6}$ full of water



Apply

11. Copy and complete the following table.

Right Rectangular Prism

	Length (cm)	Width (cm)	Height (cm)	Volume (cm ³)
a)	7	2		70
b)	12		10	1080
c)		15	5	1200

12. Copy and complete the following table.

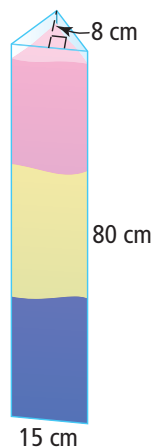
Right Triangular Prism

	Base (cm)	Height of Triangle (cm)	Height of Prism (cm)	Volume (cm ³)
a)	7	2		70
b)	18		10	1080
c)		14	5	700

13. A landscaper has 0.5 m³ of gravel to use as the base of a patio. If the gravel base must be 10 cm deep and the patio is 2.6 m wide and 2.8 m long, does she have enough gravel? How much extra gravel does she have, or how much more will she need?

The word *deep* is sometimes used to describe the height.

14. A glass vase in the shape of a right triangular prism is filled with coloured sand as a decoration in a living room. What is the volume of the vase?



15. Calculate the volume of concrete used to make a sidewalk 1.5 m wide and 120 m long. The concrete is poured 0.1 m thick.

16. Cindy's aquarium stands 75 cm tall and has a base that measures 1.2 m \times 80 cm. At one point during the initial filling, the aquarium has a 12-cm depth of water in it. Cindy needs to fill it to 15 cm from the top before she adds the fish. Draw a diagram and label the dimensions of the aquarium. Determine how much more water Cindy must add before she puts in the fish.



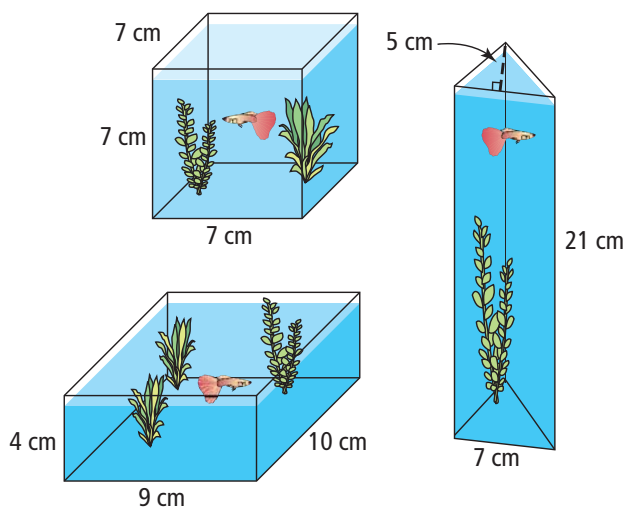
17. A contractor is excavating a rectangular hole 10 m \times 12 m \times 3 m to pour the foundation for a house. A dump truck with a capacity of 9 m³ is used to haul away the excavated soil. How many trips does the truck need to make?

Literacy Link

Capacity refers to the greatest volume that a container such as a tank, a truck, or a measuring cup can contain.

18. Suki has 30 small linking cubes.
- She wants to use 18 of them to make a large cube. Is this possible? Why or why not?
 - What number of linking cubes would she use to construct the largest cube she can possibly make?

19. Melissa has three glass vases. She wants to use one as a decorative fish tank for Harvey the guppy. Which will give Harvey the most water to swim in?



20. The ratio of length : width : height of a box is 6 : 3 : 2. What is its volume if the height is 5 cm?

21. Sketch and label all possible rectangular prisms with dimensions that are whole numbers of centimetres that have a volume of 120 cm^3 . Do not consider a change of orientation as a new figure; for example, $4 \text{ cm} \times 5 \text{ cm} \times 6 \text{ cm}$ is the same as $6 \text{ cm} \times 4 \text{ cm} \times 5 \text{ cm}$.

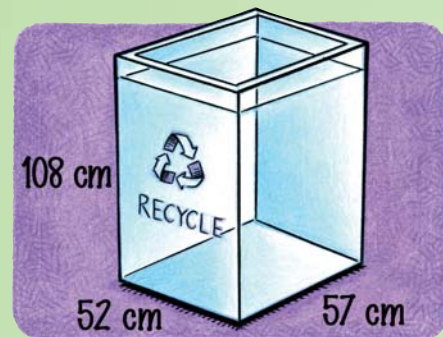
22. A tank made of concrete has outside dimensions measuring $6 \text{ m} \times 3 \text{ m} \times 1 \text{ m}$. It has no lid. The concrete is 8 cm thick. What is the maximum volume the concrete tank can hold? What assumptions did you make?

Extend

23. Rectangular Prism A and Rectangular Prism B have the same length. The width of A is half the width of B. The height of A is twice the height of B. What is the difference in volume?
24. A rectangular tank, 40 m long by 30 m wide, is filled with 960 m^3 of water.
- Determine the depth of water.
 - If the water drains out at a rate of $60 \text{ m}^3/\text{h}$, how much water is left after 2.5 h? What is the new depth of water?
 - Later, the depth of the water is 0.2 m. For how long has the tank been draining?

MATH LINK

The Parks Committee is considering putting 12 of these recycling bins throughout the park. If the bins are filled to the brim and emptied twice weekly, what volume of waste is recycled each week?



7.3

Volume of a Cylinder

Focus on...

After this lesson, you will be able to...

- determine the volume of a cylinder

Did You Know?


On average, each person in Canadian municipalities uses 604 L of water every day.

How much water do you use? You might be surprised. The water storage tank shown has a height of about 21.6 m and a diameter of about 10.5 m. If the tank is completely filled, predict how long the water would last in your city or town.



Explore the Math

Materials

- centimetre grid paper
-  a variety of empty cylindrical cans
- ruler
- measuring cups
- sand or rice

How can you use area to develop a formula for the volume of a cylinder?

Work with a partner.

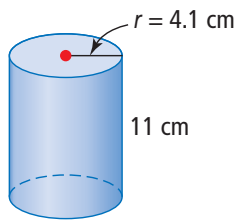
1. Choose a can. Estimate the volume of your can.
2. Calculate the volume of your can. Explain your method.
3. Share your results with three other groups. Compare the estimated and calculated volumes for each cylinder to verify the methods used.

Reflect on Your Findings

4. If you know only the radius of the base and the height of a cylinder, how can you determine the volume of the cylinder? Explain and verify your formula using an example.

Example 1: Determine the Volume of a Cylinder Given the Radius

- a) Estimate the volume of the cylinder.



- b) Calculate the volume of the cylinder. Express your answer to the nearest tenth of a cubic centimetre.

Solution

- a) Use 10 as an approximate value for the height of the cylinder.
Use 4 as an approximate value for the radius of the circular base.

Volume of a cylinder = Area of circular base \times height of cylinder

$$\begin{aligned}V &= (\pi \times r^2) \times h \\V &\approx (3 \times 4^2) \times 10 \\V &\approx 3 \times 16 \times 10 \\V &\approx 48 \times 10 \\V &\approx 480\end{aligned}$$

An estimate for the volume of the cylinder is 480 cm^3 .

- b) Volume of a cylinder = Area of circular base \times height of cylinder

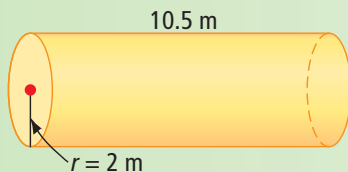
$$\begin{aligned}V &= (\pi \times r^2) \times h \\V &\approx (3.14 \times 4.1^2) \times 11 \\V &\approx 580.6174\end{aligned}$$

The volume of the cylinder is 580.6 cm^3 to the nearest tenth of a cubic centimetre.



Show You Know

- a) Estimate the volume of the cylinder.



- b) Calculate the volume of the cylinder.

Did You Know?

Vulcanization refers to a curing process for rubber that involves high heat and the addition of sulfur.

Example 2: Determine the Volume of a Cylinder Given the Diameter

A hockey puck is made of vulcanized rubber. What is the volume of rubber required to manufacture one puck? Express your answer to the nearest cubic centimetre.



Solution

The diameter is 7.6 cm.

$$\begin{aligned} r &= 7.6 \div 2 \\ &= 3.8 \end{aligned}$$

The radius is 3.8 cm.

Volume = Area of base \times height

$$V = (\pi \times r^2) \times h$$

$$V \approx (3.14 \times 3.8^2) \times 2.5$$

$$V \approx 113.354$$

The volume of the hockey puck is 113 cm³ to the nearest cubic centimetre.

How is the radius related to the diameter?

Show You Know

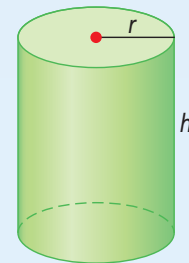
What volume of recyclable waste will fit into one of these bins?



Key Ideas

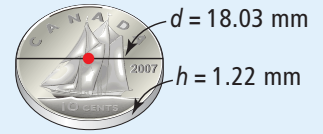
- The base of a cylinder is a circle. The formula for the area of the base of a cylinder is $A = \pi \times r^2$.
- The volume of a cylinder can be found using the formula:
Volume = Area of base \times height

$$V = (\pi \times r^2) \times h$$

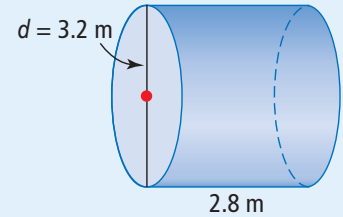


Communicate the Ideas

- List the steps you would use to find the volume of this dime.
 - What information would you need to calculate the volume of a roll of dimes?



- Hanna wants to calculate the volume of the cylinder shown but she does not know which measurement is the height.
 - What is the height?
 - Explain to Hanna how you know this is the height.



- Jethro calculated the volume of the cylinder shown.

$$V = (\pi \times r^2) \times h$$

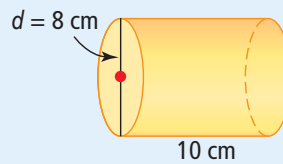
$$V \approx (3.14 \times 8^2) \times 10$$

$$V \approx 3.14 \times 64 \times 10$$

$$V \approx 2009.6$$

The volume of the cylinder is 2009.6 cm³.

Has he made an error in his solution? Explain how you know.

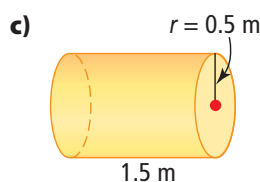
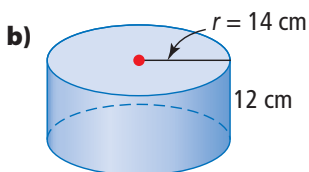
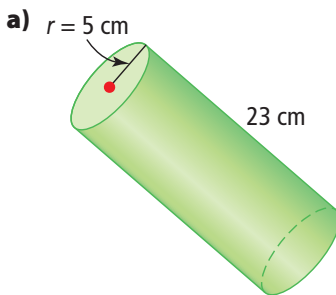


Check Your Understanding

Practise

For help with #4 to #5, refer to Example 1 on page 263.

- Determine the volume of each cylinder.

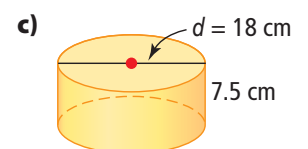
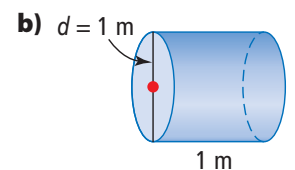
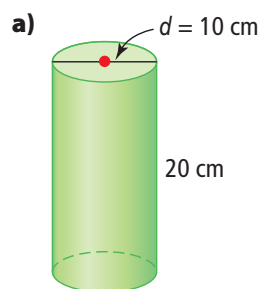


- What is the volume of each cylinder?

- radius = 5 cm, height = 8 cm
- radius = 11 cm, height = 11 cm
- radius = 1.1 m, height = 2.6 m
- height = 25 cm, radius = 4.5 cm

For help with #6 to #7, refer to Example 2 on page 264.

- Determine the volume of each cylinder.



7. What is the volume of each cylinder?
- diameter = 8 cm, height = 12 cm
 - height = 7 m, diameter = 2 m
 - height = 37.5 cm, diameter = 12 cm
 - diameter = 4.5 m, height = 19.5 m

Apply

8. The volume of a cylinder is 150 cm^3 and the area of its base is 48 cm^2 . What is the height to the nearest centimetre?
9. The Canadarm has a cylinder called a *capture envelope* that is used to catch objects floating in space. The capture envelope is 20.3 cm in diameter and 10 cm deep. What is the maximum volume of the capture envelope?



WWW Web Link

To learn more about the Canadarm, go to www.mathlinks8.ca and follow the links.

10. As of early 2006, the International Space Station consisted of several cylindrical elements.

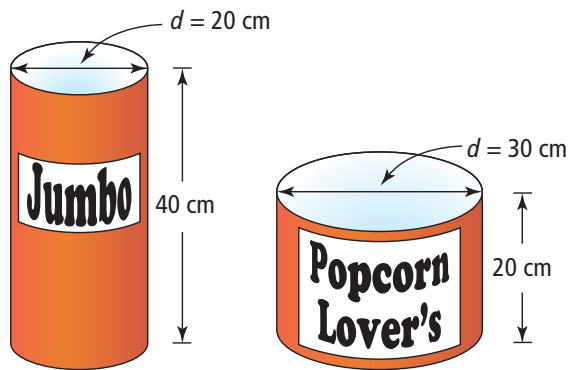
Element	Length (m)	Diameter (m)
Zarya FGB	12.6	4.1
Unity Node 1	5.5	4.6
Zvezda service module	13.1	4.2
Z1 Truss	4.9	4.2
P6 Truss solar array	73.2	10.7
Destiny	8.5	4.3

- Which element has the greatest volume? What is its volume?
- Estimate and calculate the total volume of the International Space Station.

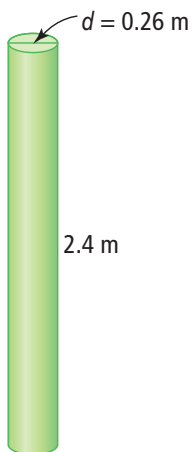
Science 8 Link

The International Space Station takes about 92 min to orbit Earth once.

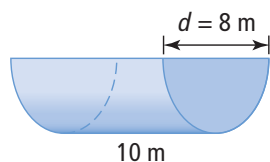
11. Some of the largest drill pipes used in extracting oil have a length of 20 m and an inside diameter of 0.5 m. As oil flows through such a pipe, what is the maximum volume of oil in one pipe at any given time?
12. Martha has a choice of two different popcorn containers at a movie. Both containers are the same price. Which container should Martha buy if she wants more popcorn for her money? Explain.



13. A company uses cardboard tubes like the one shown to make concrete posts for the foundation of a building. If a building requires 35 tubes, what is the volume of concrete required? Give your answer to the next highest cubic metre to make sure that there is enough concrete.



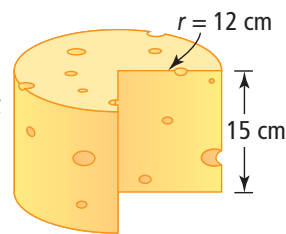
14. Determine the volume of the semi-circular trough.



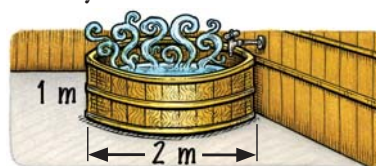
Extend

15. a) What happens to the volume of a cylinder when its radius is doubled? Show how you know.
 b) What happens to the volume of a cylinder when its height is doubled? Show how you know.

16. A piece of cheese was cut from a cylindrical block of cheddar. What volume of cheese was cut from the block? What assumptions did you make?



17. Some Japanese bathtubs are in the shape of a cylinder.



- a) Calculate the volume of water if the tub is filled to a depth of 0.6 m.
 b) If the volume of water is 1.256 m^3 , how deep is the water?
 c) If the water is already 0.5 m deep, how much more water is needed to fill it to a depth of 0.7 m?
18. A cylindrical water storage tank has a height of 21.6 m and a diameter of 10.5 m. If the tank is completely full, how long would the water last in a community of 10 000 people? Assume the average daily water use in the community is 604 L per person. Give your answer to the nearest hour.

$1000 \text{ L} = 1 \text{ m}^3$

MATH LINK

Picnic tables can have either a circular or a rectangular concrete top. The top is held up by a solid concrete column in the shape of a cylinder.

- a) Design two concrete tables.
- Each table has a column with a diameter of 60 cm and a height of your choice.
 - One table must have a circular top. The other table must have a rectangular top.
 - The tabletops cannot exceed a thickness of 10 cm.
- b) Determine the volume of concrete needed to make both picnic tables. Show your calculations.



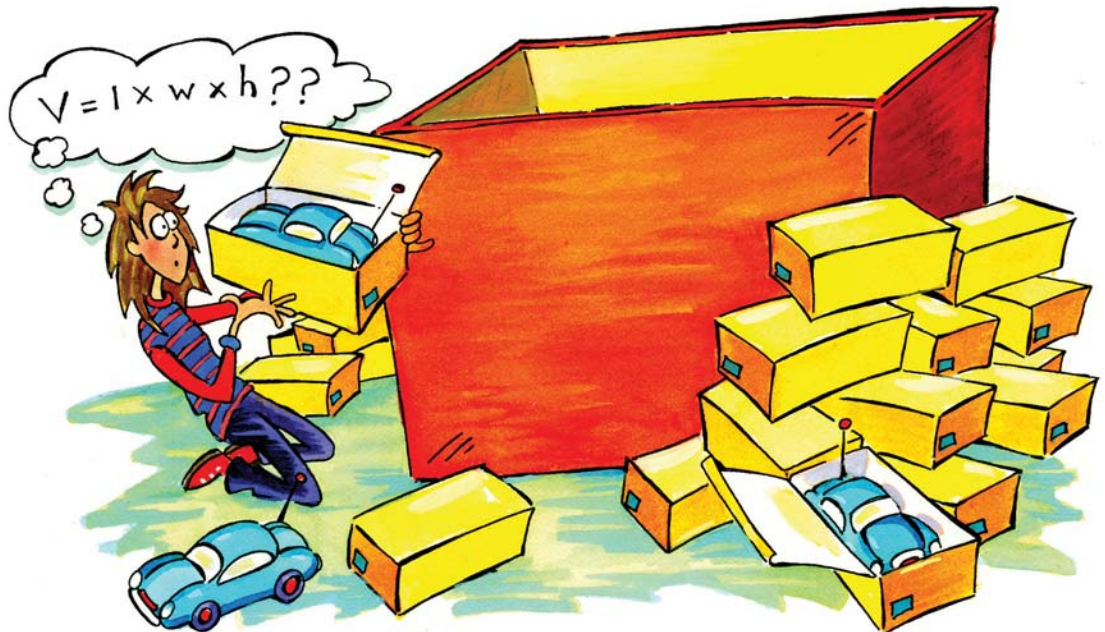
7.4

Solving Problems Involving Prisms and Cylinders

Focus on...

After this lesson, you will be able to...

- solve problems involving right rectangular prisms, right triangular prisms, and right cylinders



Danielle works at a toy store that sells remote control cars. She wants to fit 60 car boxes into a large crate. The car boxes have dimensions of $50\text{ cm} \times 30\text{ cm} \times 20\text{ cm}$. The crate has dimensions of $140\text{ cm} \times 120\text{ cm} \times 110\text{ cm}$. Predict whether all 60 boxes fit in the crate.

Explore the Math

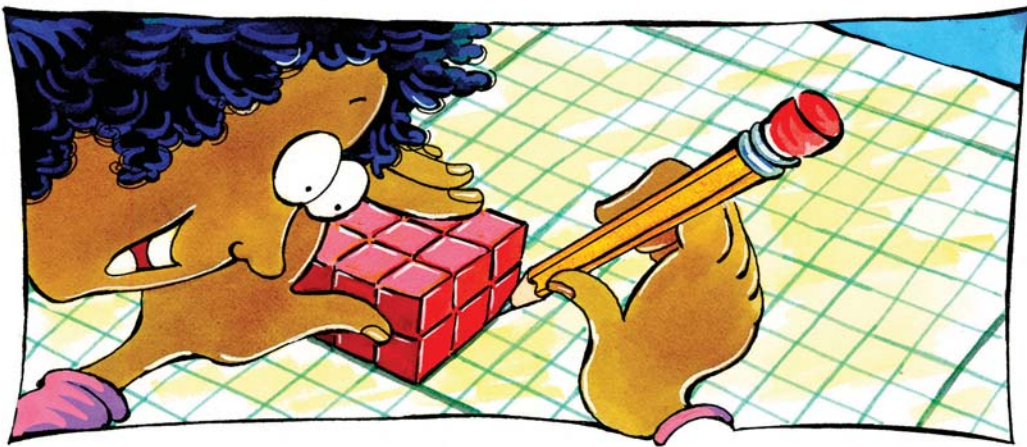
Materials

- centimetre cubes
- centimetre grid paper



How can you solve a problem involving volume?

1. Calculate the volume of one car box and the volume of the crate described above.
2. Estimate the number of boxes that could fit into the crate.
3. Model the problem to determine how many boxes you can fit in the crate.



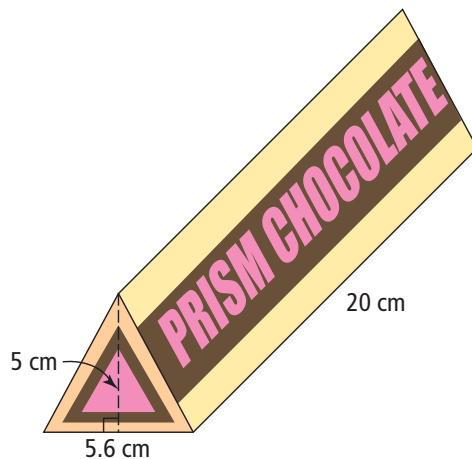
4. a) Share your model with your classmates. What was the greatest number of boxes that fit into the crate?
- b) Could you arrange your boxes differently to improve the modelled number of boxes that would fit in the crate? Explain.

Reflect on Your Findings

5. How did the estimated number of boxes compare with the modelled number of boxes that would fit in the crate? Explain any differences.

Example 1: Solve a Problem Involving Right Triangular Prisms

Marcus is making a display of packages of Prism Chocolates in his candy shop. He will stack 64 packages to form a shape that is a triangular prism, using eight packages in the bottom layer. What is the volume of the display? Show your thinking.



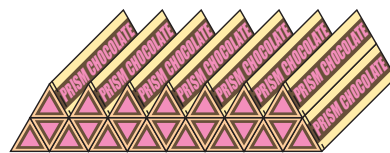
Strategies

Draw a Diagram

Solution

The packages are triangular prisms.

The best way to stack the packages is to place Layer 1 on the table, then invert Layer 2 in the cavities between the packages in the first layer. In order to maintain a triangular shape, Layer 3 must have the same number of packages as Layer 2.



Determine the volume of one package:

Volume = (base of triangle \times height of triangle \div 2) \times height of prism

$$V = (5.6 \times 5 \div 2) \times 20$$

$$V = 14 \times 20$$

$$V = 280$$

The volume of one package is 280 cm³.

The number of packages used in the display is 64.

$$\begin{aligned} \text{The volume of the display} &= 280 \times 64 \\ &= 17\,920 \end{aligned}$$

The volume of the display is 17 920 cm³.

Example 2: Solve a Problem Involving Cylinders

A cylinder with a radius of 0.6 m and a height of 15 m needs to be replaced with a cylinder of equal volume. However, the new cylinder has a radius of 0.5 m. How high must the new cylinder be?

Solution

Determine the volume of the original cylinder.

$$V = \pi \times r^2 \times h$$

$$V \approx 3.14 \times 0.6^2 \times 15 \quad \boxed{C} \quad 3.14 \times .6 \times .6 \times 15 = 16.956$$

$$V \approx 16.956$$

The original cylinder has a volume of 16.956 m³.

To determine the new height, replace all variables in the formula with values except for h .

$$V = \pi \times r^2 \times h$$

$$16.956 \approx 3.14 \times 0.5^2 \times h$$

$$16.956 \approx 0.785h$$

Divide both sides of the equation by 0.785 to isolate the variable.

$$\frac{16.956}{0.785} \approx \frac{0.785}{0.785} h \quad \boxed{C} \quad 16.956 \div .785 = 21.6$$

$$21.6 \approx h$$

The radius is now 0.5 m.
The volume of 16.956 m³
is the same.

The new cylinder must have a height of 21.6 m to contain the same volume as the original cylinder.

Strategies

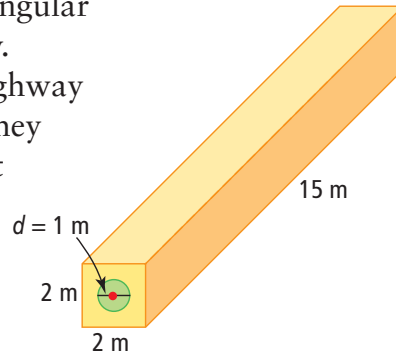
Solve an Equation

Show You Know

Workers must replace a cylindrical pipe with a radius of 0.4 m and a length of 12 m. The new pipe has a radius of 0.6 m. The volume must remain the same. How long must the new pipe be?

Example 3: Solve a Problem Involving Right Prisms and Cylinders

Engineers Rob and Kyla have designed rectangular culverts to carry water under a new highway. They estimate that the distance under the highway is 45 m. Determine the volume of concrete they need to make the required number of culvert pieces. Give your answer to the next highest tenth of a cubic metre.



Solution

Draw a diagram of the culvert under the highway.

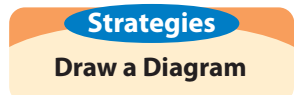
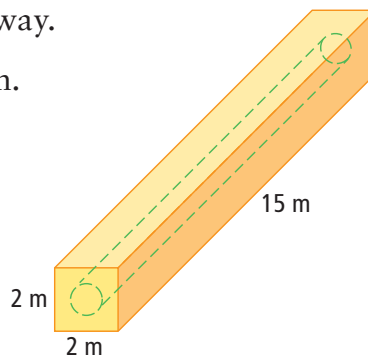
Determine the volume of the rectangular prism.

$$V = l \times w \times h$$

$$V = 2 \times 2 \times 15$$

$$V = 60$$

The volume of the rectangular prism is 60 m³.



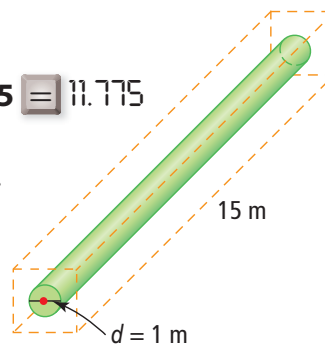
Determine the volume of the cylindrical space.

$$V = (\pi \times r^2) \times h$$

$$V \approx 3.14 \times 0.5^2 \times 15 \quad \mathbf{C} \quad 3.14 \times .5 \times .5 \times 15 = 11.775$$

$$V \approx 11.775$$

The volume of the cylindrical space is 11.775 m³.



$$\begin{aligned} \text{Volume of concrete required} &= \text{volume of prism} - \text{volume of cylindrical space} \\ &\approx 60 - 11.775 \\ &\approx 48.225 \end{aligned}$$

The volume of concrete required for one culvert piece is 48.225 m³.

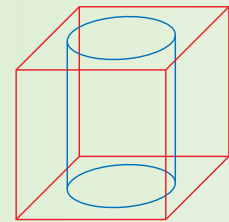
Determine how many culvert pieces Rob and Kyla will need.
 The distance under the highway is 45 m. The length of each culvert is 15 m.
 $45 \div 15 = 3$
 They will need three culvert pieces.

Calculate the volume of concrete required for three culvert pieces.
 $3 \times 48.225 = 144.675$

The volume of concrete required for three culvert pieces is 144.7 m³ to the nearest tenth of a cubic metre.

Show You Know

A cube has edges 40 cm long. A cylindrical section with a radius of 15 cm is removed from the cube. What is the remaining volume of the cube, to the nearest tenth of a cubic metre?

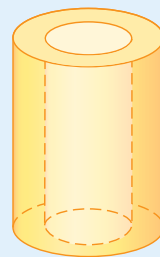
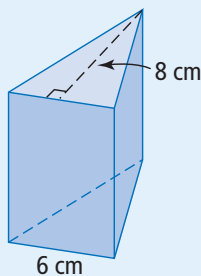


Key Ideas

- There are different types of problems involving volumes of prisms and cylinders.
 - You may need to decide which formula to use.
 - It may help to draw a diagram.
- Some problems may involve more than one set of calculations.

Communicate the Ideas

1. The triangular prism shown has a volume of 264 cm³. Explain how you could find its height.
2. The object shown is hollow. Explain how you would determine its volume.

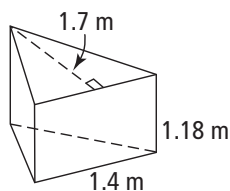


Check Your Understanding

Practise

For help with #3, refer to Example 1 on pages 269–270.

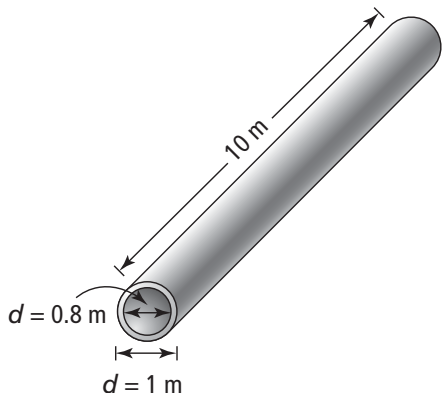
3. An artist has 20 triangular prisms like the one shown. He decides to use them to build a giant triangular prism with a triangular base of length 5.6 m and height 6.8 m.



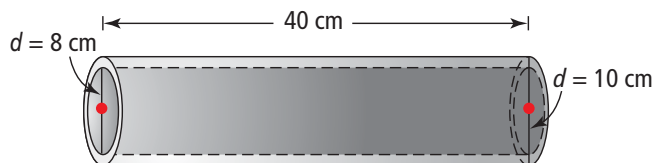
- Does he have enough small prisms?
- What is the volume of the new prism to the nearest hundredth of a metre?

For help with #4 to #6, refer to Example 2 on pages 270–271.

4. Two cylinders have the same volume. The first cylinder has a diameter of 10 cm and a height of 30 cm. The second cylinder has a diameter of 8 cm. What is the height of the second cylinder, to the nearest tenth of a centimetre?
5. A concrete culvert that is 10 m long has an outside diameter of 1 m and an inside diameter of 0.8 m. Determine the volume of concrete required to make the culvert, to the nearest tenth of a cubic centimetre.

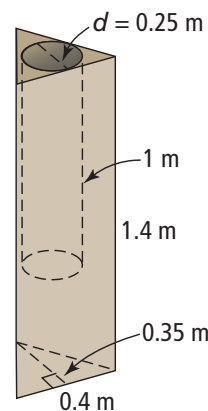


6. A pipe has an outside diameter of 10 cm, an inside diameter of 8 cm, and a height of 40 cm. What is the capacity of the pipe, to the nearest tenth of a cubic centimetre?



For help with #7, refer to Example 3 on pages 271–272.

7. A clay planter has the shape of a right triangular prism as shown. Inside the planter is a cylindrical hole. Calculate the volume of clay needed to make the planter, to the nearest tenth of a cubic centimetre.



Apply

8. Manuel's company uses shipping crates with dimensions 3 m × 3 m × 7 m. He has to ship 25 000 boxes with dimensions 10 cm × 10 cm × 20 cm. Calculate whether one crate will be enough.
9. Laura, an office manager, has purchased a carton that is 300 cm × 400 cm × 600 cm to store 9000 boxes of files. Each box has dimensions 30 cm × 26 cm × 10 cm. Calculate whether all of the files will fit in the carton.

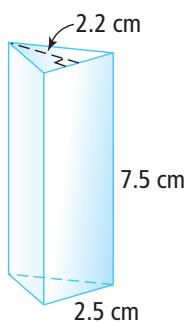
10. In the cafeteria at Prairietown School, the garbage can is filled up twice every lunch hour. The garbage can is a cylinder with a radius of 25 cm and a height of 95 cm.

- Determine the volume of garbage produced each day in the cafeteria.
- Determine the volume of garbage produced in a 5-day week.
- The school's environment club wants to reduce the weekly garbage to below $470\,000\text{ cm}^3$ by encouraging students to recycle. To reach this goal, how many times should the garbage can be filled each lunch hour?

11. A cylinder has a diameter of 80 cm and a length of 45 cm. Another cylinder has the same volume but is 80 cm long. What is the diameter of the longer cylinder?

12. A rectangular tub with dimensions $2\text{ m} \times 1\text{ m} \times 0.5\text{ m}$ is filled with water using a pail of radius 0.1 m and height 0.35 m. How many pails of water will be required? Give your answer to the nearest whole pail.

13. A manufacturer makes right triangular prisms like the one shown for refracting light. They will be packed in boxes 12.5 cm long, 2.5 cm wide, and 22.5 cm high. How many prisms can fit in a box?



Science Link

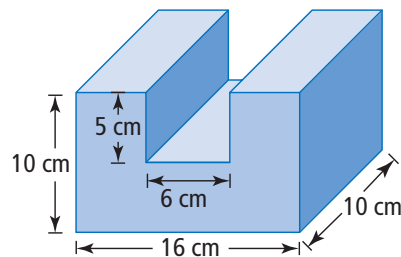
When a prism refracts light, it divides light into the colours of the spectrum.



14. Ted sells his homemade peanut butter for \$1.60 a jar at the local Farmers' Market. The jar is 8 cm in diameter and 10 cm high. He decides he will also sell peanut butter in jars that are 16 cm in diameter and 20 cm high. What should he charge if he uses the same price per cubic centimetre?

15. a) A wooden block is formed in the shape shown by cutting a right rectangular solid from a larger one. What is the volume of the solid shown?

b) Check your calculations by using a second method to solve the problem.



16. Fatima wants to fill a circular wading pool. She does not have a hose, so she uses a rectangular pail that she fills from a tap. The inside diameter of the pool is 120 cm and it is 25 cm deep. The inside dimensions of the pail are $30\text{ cm} \times 22\text{ cm} \times 24\text{ cm}$ deep.



a) Fatima wants to fill the pool to a depth of 18 cm. What volume of water does she have to carry?

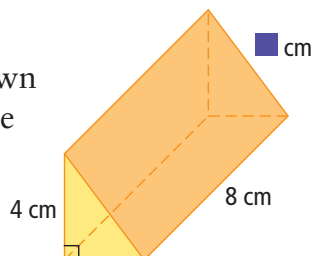
b) Each time she goes to the tap, Fatima fills the pail to a height of 20 cm. What is the volume of water in the pail?

c) Calculate how many pails of water Fatima has to carry to fill the pool to a depth of 18 cm.

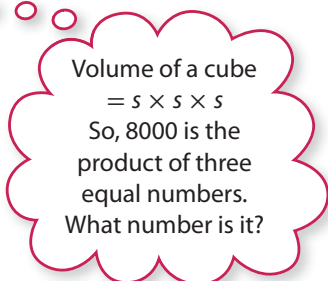
17. A sheet of paper that is 22 cm by 28 cm can be used to make a cylinder by rolling it in two different ways. Which way produces the larger volume? Show your work.

Extend

18. The volume of the triangular prism shown is 48 cm^3 . What is the value of the missing measurement? Show your work.

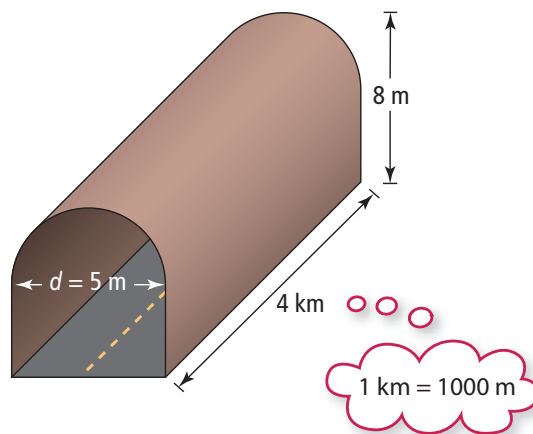


19. A cylindrical vase fits perfectly in a cube-shaped box. If the box has a volume of 8000 cm^3 , what is the volume of the vase?



20. Kevin and Jasjot plan to install a culvert that is 8 m long and holds a volume of 40 m^3 of water. What diameter of culvert should they use?

21. The end of a car tunnel has the shape of a semi-circle on top of a rectangle. The tunnel is exactly 4 km long.
- Calculate the volume of air in the tunnel with no cars in it.
 - The air in a car tunnel must be exchanged frequently. If the exhaust system pumps the air out at a rate of 10 m^3 per second, how long does it take to replace the stale air with fresh air in the entire tunnel? Give your answer in hours and minutes.



MATH LINK

Shrub and flower planters have a variety of shapes. Some of the shapes could be connected to create a more interesting appearance.

- Design two different planters. One must be a right triangular prism.
- If the walls of the planters are 7 cm thick, determine the volume of concrete needed to construct one of your planters.
- What volume of dirt do you need to fill the planter from part b) to 2 cm from the top?

