

# 9

## Add and Subtract Integers

Canada's national winter sport is ice hockey. A game like modern hockey was first played in Montréal in 1875. Over 20 countries now compete internationally, but Canada still produces many of the world's best hockey players.

In this chapter you will learn about a connection between hockey and integers. This connection is called the plus/minus score. Each year, the player with the best plus/minus score in the NHL wins an award. Bobby Orr won it six times. Wayne Gretzky won it four times.

### WWW Web Link

Canada has two official national sports: hockey in the winter and lacrosse in the summer. To find out more about lacrosse, go to [www.mathlinks7.ca](http://www.mathlinks7.ca) and follow the links. Who developed the game of lacrosse? What was the game originally called?

### What You Will Learn

- to add and subtract integers using concrete materials, diagrams, and symbols

### Key Words

zero pair  
opposite integers

### MATH LINK

Suggest ways of measuring a hockey player's performance. Would you use the same measure for players in different positions? For example, would you measure a goalie's performance in the same way as a forward's performance?

In this chapter, you will learn how to calculate and use the plus/minus score to measure a hockey player's contribution to a team.





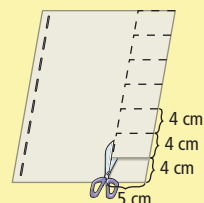
**Make the following Foldable to organize what you learn in Chapter 9.**

**Step 1** Staple eight sheets of notebook paper together along the left edge.

**Step 2** Make a mark 4 cm up from the bottom of the top page. Cut through the top seven sheets about 5 cm in from the right edge up to this mark and across to the right edge. Do not cut the last page, which is your back cover.

**Step 3** Cut through the top six sheets of paper, up 4 cm more and across to the right edge. As you do this, you will form tabs along the right side of the foldable.

**Step 4** Continue to cut tabs in this way until you have seven tabs.



**Step 5** Label the tabs as shown in the diagram.

	Key Words
Chapter 9	9.1
Adding and Subtracting	9.2
Integers	9.3
	9.4
	9.5
	What I Need to Work On

### Literacy Link

As you work through Chapter 9, take notes under the appropriate tab. Include information about the key words, examples, and key ideas.

# 9.1

## Explore Integer Addition

### Focus on...

After this lesson, you will be able to...

- add integers using integer chips

### Did You Know?

Canada has the longest coastline of any country. About 7% of Canada's coastline is in British Columbia. Over 7000 species of plants and animals have been identified in British Columbia's coastal waters.

Dr. Verena Tunnicliffe of the University of Victoria is an expert on life in the sea. She directs the Victoria Experimental Network Under the Sea project (VENUS). Scientists use underwater detectors located between Vancouver Island and mainland British Columbia to show conditions from the sea floor to the surface. The detectors also monitor the movements of animals, such as whales and fish.



The first VENUS detectors were installed in Saanich Inlet and the Strait of Georgia. The maximum depth of Saanich Inlet is 230 m. The Strait of Georgia is 170 m deeper than that. What is the maximum depth of the Strait of Georgia?

### Explore the Math


#### How can you use integer chips to add two integers?

Integer chips are coloured disks that represent integers. In this book, a red chip represents  $+1$ , and a blue chip represents  $-1$ . A **zero pair** includes one red chip and one blue chip.

### Materials

- red and blue integer chips

#### zero pair

- a pair of integer chips, with one chip representing  $+1$  and one chip representing  $-1$
- 
  
 $+1$       $-1$
- the pair represents zero because  $(+1) + (-1) = 0$

- Use red chips to model  $+2$ .
  - Use blue chips to model  $-2$ .
  - Combine the two sets of chips and organize them into zero pairs. What is the sum of  $+2$  and  $-2$ ? How do you know?
  - Copy and complete the addition statement  $(+2) + (-2) = \blacksquare$ .
- Repeat #1 for the integers  $+3$  and  $-3$ .

3. a) Predict the sum of any pair of **opposite integers**.  
Explain your reasoning.
- b) Use integer chips to check the predicted sum of  $+6$  and  $-6$ .
- c) Copy and complete the addition statement  $(+6) + (-6) = \blacksquare$ .
4. a) Model  $+5$  and  $-3$ .
- b) Combine the two sets of chips. Organize them into as many zero pairs as possible. How many chips are left over? What colour are they?
- c) What is the sum of  $+5$  and  $-3$ ? How do you know?
- d) Copy and complete the addition statement  $(+5) + (-3) = \blacksquare$ .
5. Repeat #4 for the integers  $+3$  and  $-5$ .
6. Use integer chips to add each of the following. Write the addition statement in each case.
- a)  $+2$  and  $+3$     b)  $-1$  and  $-5$     c)  $+4$  and  $-2$     d)  $+3$  and  $-6$

## Reflect on Your Findings



7. Describe how you can use integer chips to add two integers.

## Example 1: Add Integers Using Integer Chips

Add using integer chips. Copy and complete the addition statement in each case.

- a)  $(+1) + (+2) = \blacksquare$                       b)  $(-3) + (-2) = \blacksquare$   
 c)  $(+5) + (-4) = \blacksquare$                       d)  $(-6) + (+2) = \blacksquare$

### Solution

- a)  $+1$ :   
 $+2$ : 

There are three red chips altogether.

The sum is  $+3$ .

The addition statement is  $(+1) + (+2) = +3$ .

- b)  $-3$ :   
 $-2$ : 

There are five blue chips altogether.

The sum is  $-5$ .

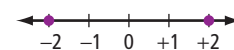
The addition statement is  $(-3) + (-2) = -5$ .

## opposite integers

- two integers with the same numeral, but different signs. For example,



- two integers represented by points that are the same distance in opposite directions from zero on a number line



## Literacy Link

### Writing and Reading Integer Sums

The  $+$  or  $-$  sign of an integer does not represent addition or subtraction. To avoid confusion, use brackets to distinguish integer signs from operation symbols. For example, write  $(+2) + (-2)$ , *not*  $+2 + -2$ . You may find it helpful to read  $(+2) + (-2)$  as “positive two plus negative two.”

### Strategies

#### Model It

Refer to page xvi.

### Strategies

What other strategy could you use?

c) +5:   
 -4: 

When the chips are paired up, there is one red chip left over.  
 Each zero pair represents 0, so the sum is +1.  
 The addition statement is  $(+5) + (-4) = +1$ .

d) -6:   
 +2: 

When the chips are paired up, there are four blue chips left over.  
 Each zero pair represents 0, so the sum is -4.  
 The addition statement is  $(-6) + (+2) = -4$ .

### Show You Know

Add using integer chips. Show your thinking with a diagram.

a)  $(-3) + (-3)$

b)  $(+4) + (+6)$

c)  $(-1) + (+3)$

d)  $(+3) + (-8)$

### Example 2: Apply Integer Addition

Ray spent \$5 on bus fares to visit his aunt. During the visit, she gave Ray \$12 for doing some yard work. How much money did Ray gain from the visit?

#### Solution

Represent the \$5 Ray spent by a negative integer, -5.

Represent the \$12 Ray received by a positive integer, +12.

The money that Ray gained from the visit can be represented by the sum  $(-5) + (+12)$ .

Add  $(-5) + (+12)$  using integer chips.



$$(-5) + (+12) = +7$$

Ray gained \$7 from the visit.

Ray got more than he spent, so the sum should be positive.

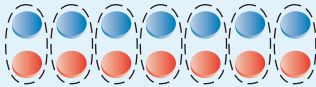


## Key Ideas

- You can use integer chips to represent integer addition.
- A zero pair, which includes one +1 chip and one -1 chip, represents 0.
- The sum of any two opposite integers is zero.



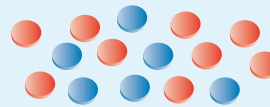
$$(-7) + (+7) = 0$$



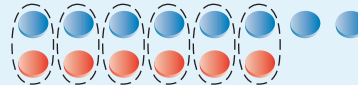
-7 and +7 are opposite integers.

## Communicate the Ideas

- Do the integer chips in the diagram represent a sum of +3 or -3? How do you know?



- What addition statement do the integer chips in the diagram represent? Explain your reasoning.



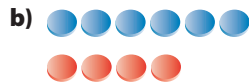
- Suppose that the sum of two integers is represented by equal numbers of red and blue chips. Can you state the sum without knowing how many chips there are? Explain.

- David asked his classmate Avril to show him why  $(+1) + (-1) = 0$ . She modelled the addition by climbing up one step and then climbing down it again. Explain how her model shows that  $(+1) + (-1) = 0$ .

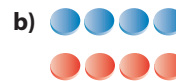
## Practise

For help with #5 to #8, refer to Example 1 on page 311–312.

- What addition statement does each diagram represent?



- What addition statement does each diagram represent?



7. Add using integer chips. Have a partner check your chips. Then copy and complete the addition statement.

- a)  $(+3) + (+4) = \blacksquare$
- b)  $(-2) + (-4) = \blacksquare$
- c)  $(+5) + (-2) = \blacksquare$
- d)  $(-8) + (+8) = \blacksquare$

8. Add using integer chips. Then copy and complete the addition statement.

- a)  $(-4) + (-1) = \blacksquare$
- b)  $(+2) + (+6) = \blacksquare$
- c)  $(-7) + (+4) = \blacksquare$
- d)  $(+8) + (-3) = \blacksquare$

**Apply**

For help with #9 to #12, refer to Example 2 on page 312.

- 9. Use the sum of two integers to represent each situation.
  - a) Sharon found \$10 and then lost \$4. How much did she have left?
  - b) A snail slid 7 cm down a stalk and climbed 5 cm back up. How far was the snail below its original position?
  - c) In one game, the Rockies girls' soccer team scored 4 goals and had 1 goal scored against it. How many goals did the team win by?
  - d) A scuba diver dove 4 m under the water and then went down another 8 m. What was the diver's final depth under the water?
- 10. Miguel spent \$6 on Saturday morning and another \$9 on Saturday afternoon. How much less money did he have at the end of the day than at the beginning? Use integer addition to determine your answer.

11. The temperature on the Moose Lake Reserve in Manitoba was  $+6^{\circ}\text{C}$ . The temperature dropped by  $10^{\circ}\text{C}$  to reach the overnight low temperature. What was the overnight low temperature? Use integer addition to determine your answer.

**Did You Know?**

The Celsius temperature scale is named after Anders Celsius (1701–1744), a Swedish astronomer. In 1742, he divided the temperature difference between the freezing point and boiling point of fresh water into  $100^{\circ}$ . However, his scale was upside down. It had  $0^{\circ}$  at the boiling point and  $100^{\circ}$  at the freezing point. Two years later, a Swedish botanist named Carl Linnaeus (1707–1778) switched these values.

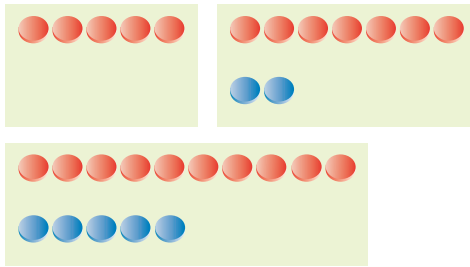
- 12. Use the sum of two integers to represent each situation. What is each sum? Explain the meaning of each numerical answer.
  - a) Nadia had 6 world-music CDs and then bought another 2 world-music CDs.
  - b) The temperature went down by  $5^{\circ}\text{C}$  and then went up by  $8^{\circ}\text{C}$ .
  - c) Parminder took 4 steps forward and 4 steps backward.
  - d) Joe caught 6 char in his net, but 2 got away as he pulled the net in.

13. a) Copy and complete the table.

$(+2) + (+3) = \blacksquare$	$(+3) + (+2) = \blacksquare$
$(-1) + (-4) = \blacksquare$	$(-4) + (-1) = \blacksquare$
$(+2) + (-2) = \blacksquare$	$(-2) + (+2) = \blacksquare$
$(+4) + (-7) = \blacksquare$	$(-7) + (+4) = \blacksquare$

- b) Compare the two addition statements on each row of the completed table. What can you conclude about the order in which you can add two integers? Test your conclusion on some other integer additions.

14. a) How do these three diagrams show three different ways to model  $+5$ ?



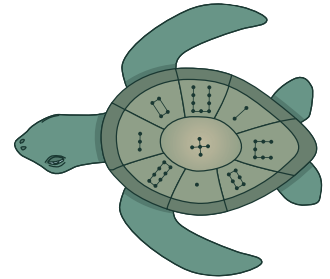
- b) Show three different ways to model  $-4$  using integer chips.
15. Identify an integer that does not have an opposite. Explain.
16. What is the opposite of the opposite of  $-3$ ? Explain.

### Extend

17. What is each sum? Use integer chips to help determine your answers.
- $(+3) + (+1) + (+2)$
  - $(-2) + (-3) + (-2)$
  - $(+2) + (-4) + (+3)$
  - $(-1) + (+4) + (-5)$
  - $(-3) + (-4) + (+1) + (-5)$
  - $(+5) + (+4) + (-3) + (-6)$
18. a) What integer is 8 more than its opposite?  
 b) Is there an integer that is 5 more than its opposite? Explain.
19. In a magic square, the numbers in each row, column, and diagonal have the same sum. This is called the magic sum. A Chinese legend from 3000 years ago describes how a child saw the first magic square in markings on a turtle's back.

- a) Describe how the markings on the turtle represent the numbers in the magic square shown beside it. What is the magic sum?

4	9	2
3	5	7
8	1	6



- b) Verify that the following square is a magic square. What is the magic sum?

$-4$	$+1$	$0$
$+3$	$-1$	$-5$
$-2$	$-3$	$+2$

- c) Add  $+2$  to each integer in the square from part b). Is the result a magic square? If so, what is the magic sum?
- d) Using the magic square from part b), create a magic square with a magic sum of  $-6$ . Explain your reasoning.
20. This diagram shows the points scored when a dart strikes each area of the target.
- 
- If one dart hits the target, what scores are possible?
  - If two darts hit the target, what total scores are possible?
  - If three darts hit the target, what total scores are possible?
  - Choose and justify a score for a dart that misses the target.
  - Use your answer from part d) to find the possible total scores when two darts hit the target and one dart misses.



# 9.2

## Add Integers

### Focus on...

After this lesson, you will be able to...

- add integers using a number line

### Did You Know?

Settlers named the warm winds chinooks because they blew from the land of the Chinook First Nation in the Pacific Northwest.

### Materials

- red and blue integer chips



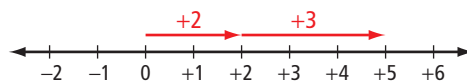
A distinctive arch of cloud appears in the western sky when a warm chinook wind approaches southern Alberta.

One January night, the temperature in Pincher Creek was  $-19^{\circ}\text{C}$ . A chinook wind increased the temperature by  $22^{\circ}\text{C}$  in one hour. Describe how you could use integer chips to determine the temperature at the end of that hour. If you did not have enough integer chips, how could you determine the temperature?

### Explore the Math

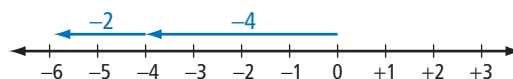
#### How can you use a number line to add two integers?

- The diagram shows how you can model  $(+2) + (+3)$  using a number line.



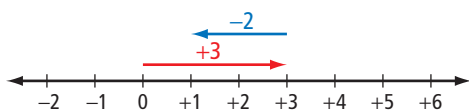
- How are the two positive integers shown in the diagram?
- Model  $(+2) + (+3)$  using integer chips. What is the sum?
- How does the number line show the sum?

- The diagram shows how you can model  $(-4) + (-2)$  using a number line.

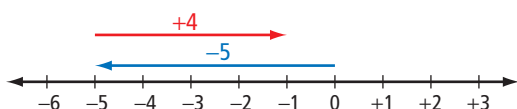


- How are the two negative integers shown in the diagram?
- Model  $(-4) + (-2)$  using integer chips. What is the sum?
- How does the number line show the sum?

3. The diagram shows how you can model  $(+3) + (-2)$  using a number line.



- a) Model  $(+3) + (-2)$  using integer chips. What is the sum?  
 b) How does the number line show the sum?
4. The diagram shows how you can model  $(-5) + (+4)$  using a number line.



- a) Model  $(-5) + (+4)$  using integer chips. What is the sum?  
 b) How does the number line show the sum?
5. Model each addition using a number line. What is the sum?
- a)  $(+3) + (-3)$                       b)  $(-2) + (-5)$   
 c)  $(+4) + (+3)$                       d)  $(+1) + (-4)$

## Reflect on Your Findings

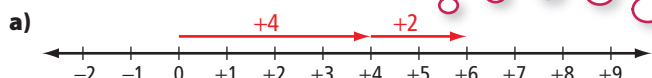
6. Describe how you can use a number line to add two integers.

## Example 1: Add Integers Using a Number Line

Add using a number line. Copy and complete the addition statement in each case.

- a)  $(+4) + (+2) = \blacksquare$                       b)  $(-5) + (-3) = \blacksquare$   
 c)  $(+2) + (-4) = \blacksquare$                       d)  $(-6) + (+6) = \blacksquare$

### Solution



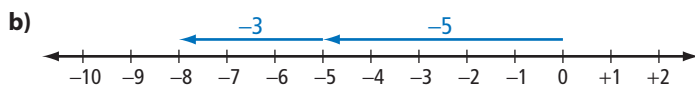
The sum is  $+6$ .

The addition statement is  $(+4) + (+2) = +6$ .

Start the first arrow at zero. Start the second arrow where the first arrow ends. The head of the second arrow is at  $+6$ .

#### Strategies

**Draw a Diagram**  
Refer to page xvi.

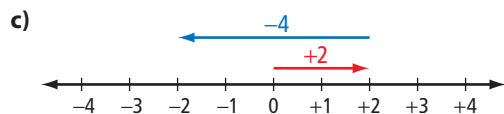


The sum is  $-8$ .

The addition statement is  $(-5) + (-3) = -8$ .

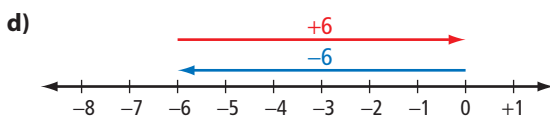
#### Strategies

What other strategy could you use?



The sum is  $-2$ .

The addition statement is  $(+2) + (-4) = -2$ .



The sum is  $0$ .

The addition statement is  $(-6) + (+6) = 0$ .

+6 and -6 are opposite integers. Their sum is 0.

### Show You Know

Add using a number line. Copy and complete the addition statement in each case.

a)  $(-4) + (-2) = \blacksquare$

b)  $(+1) + (+5) = \blacksquare$

c)  $(-2) + (+6) = \blacksquare$

d)  $(+3) + (-4) = \blacksquare$

### Literacy Link

#### Describing Temperatures

Temperatures are described in different ways. For example, the scientific way to read  $-4^{\circ}\text{C}$  is "negative four degrees Celsius." You may also hear  $-4^{\circ}\text{C}$  described in everyday speech as "minus four degrees Celsius," "four degrees below zero," or "four below."

### Literacy Link

#### Writing Positive Integers

Positive integers are usually written without the positive sign. In Example 2, you could write that the high temperature was  $5^{\circ}\text{C}$ .

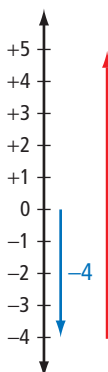
### Example 2: Apply Integer Addition

The temperature in Yellowknife, Northwest Territories, one morning in October was  $-4^{\circ}\text{C}$ . The temperature increased by  $9^{\circ}\text{C}$  to reach the high temperature for that day. What was the high temperature?

#### Solution

Calculate  $(-4) + (+9)$  to find the high temperature.

Represent the addition using a vertical number line, which looks like one type of thermometer.



On a vertical number line, positive integers are above 0, negative integers are below 0.

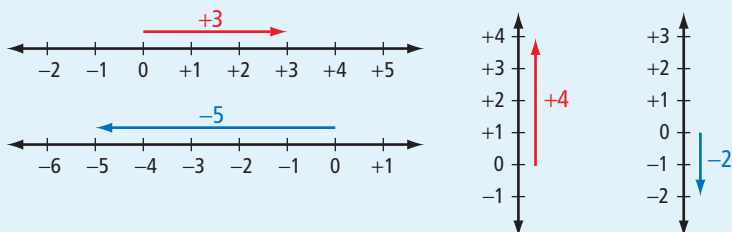
The temperature went up by  $9^{\circ}\text{C}$  from  $-4^{\circ}\text{C}$ , so the sum is positive.

$(-4) + (+9) = +5$

The high temperature was  $+5^{\circ}\text{C}$ .

## Key Ideas

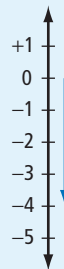
- You can use an arrow on a horizontal or vertical number line to represent an integer. The direction of the arrow shows the sign of the integer. The length of the arrow shows the value of the numeral in the integer.



- You can use a horizontal or vertical number line to represent integer addition.

## Communicate the Ideas

1. What integer does the diagram shown to the right represent? Explain.



2. Suppose that two arrows are used to represent any two opposite integers on a number line.
  - a) How do the directions of the two arrows compare? Explain.
  - b) How do the lengths of the two arrows compare? Explain.
3. What addition statement does the diagram represent? Explain how you know each term in the statement.

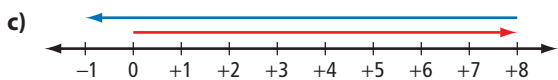
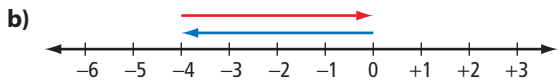
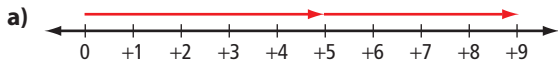


4. Jeffrey stated that the sum of  $-3$  and  $+5$  is  $-2$ . Is Jeffrey correct? Explain using a number line.

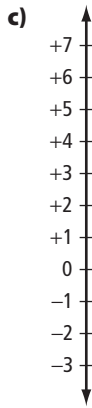
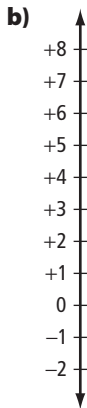
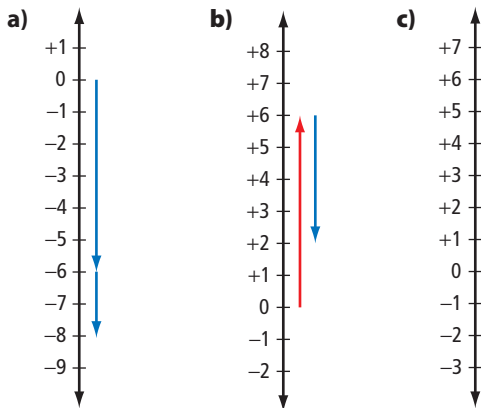
## Practise

For help with #5 to #8, refer to Example 1 on page 317–318.

5. What addition statement does each diagram represent?



6. What addition statement does each diagram represent?



7. Add using a number line. Copy and complete the addition statement.

a)  $(+5) + (+5) = \blacksquare$

b)  $(-3) + (-6) = \blacksquare$

c)  $(+4) + (-10) = \blacksquare$

d)  $(-7) + (+12) = \blacksquare$

8. Add using a number line. Copy and complete the addition statement.

a)  $(-4) + (+8) = \blacksquare$

b)  $(-4) + (-6) = \blacksquare$

c)  $(+5) + (-9) = \blacksquare$

d)  $(+10) + (-8) = \blacksquare$

## Apply

For help with #9 to #12, refer to Example 2 on page 318.

9. Kowmuk's dad gave him \$20. Kowmuk spent \$12 at the store. How much did he have left? Use a number line to help determine your answer.
10. The temperature in Resolute, Nunavut, one afternoon in May was  $-8^{\circ}\text{C}$ . The temperature decreased by  $6^{\circ}\text{C}$  to reach the overnight low temperature. What was the overnight low temperature? Use a number line to help determine your answer.
11. a) The most rapid temperature increase caused by a chinook took place in Spearfish, South Dakota. One January morning, the temperature was  $-20^{\circ}\text{C}$ . The temperature then increased by  $27^{\circ}\text{C}$  in two minutes. What was the final temperature?
- b) Later that morning, the temperature in Spearfish had climbed to  $+12^{\circ}\text{C}$ . It then dropped by  $32^{\circ}\text{C}$  in 27 minutes. What was the final temperature?
12. a) A submarine was cruising at a depth of 50 m. The captain brought the submarine up by 34 m to reach periscope depth. What was periscope depth for this submarine?



- b) The captain took the submarine down by 74 m from periscope depth to a new cruising depth. What was this cruising depth?

13. What is the sum of  $+1987$  and  $-1987$ ?  
How do you know?

14. The sum of two integers is  $-1$ .

- a) What could the integers be?  
Give four possible answers.
- b) Are there more possible answers? Explain.

15. a) Copy and complete the pattern.

$$(+9) + (-5) = \blacksquare$$

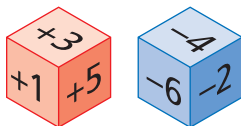
$$(+8) + (-4) = \blacksquare$$

$$(+7) + (-3) = \blacksquare$$

$$(+6) + (-2) = \blacksquare$$

- b) Describe and explain the pattern.
- c) What are the next three lines of the pattern?

16. The integers from  $+1$  to  $+6$  are marked on the faces of a number cube. The integers from  $-1$  to  $-6$  are marked on the faces of another number cube. What possible totals can you score by rolling both number cubes?



17. Suppose a friend knows how to add positive integers but has never added negative integers.

- a) How could you use the following pattern to show your friend how to add  $+6$  and  $-2$ ?

$$(+6) + (+3) = +9$$

$$(+6) + (+2) = +8$$

$$(+6) + (+1) = +7$$

$$(+6) + 0 = +6$$

$$(+6) + (-1) = \blacksquare$$

$$(+6) + (-2) = \blacksquare$$

- b) Make up a pattern to show your friend how to add  $+2$  and  $-4$ .
- c) Make up a pattern to show your friend how to add  $-3$  and  $-5$ .

18. Is each statement always true, sometimes true, or never true? Explain your reasoning.

- a) The sum of two negative integers is a positive integer.
- b) The sum of a positive integer and a negative integer is zero.
- c) The sum of a positive integer and zero is a positive integer.
- d) The sum of a positive integer and a negative integer is a negative integer.

19. Suppose you had to choose one way to model each of the following additions. Would you choose integer chips or a number line? Explain.

- a)  $(-6) + (+4)$
- b)  $(+90) + (-140)$

### Extend

20. Use a number line to add the following integers. Use integer chips to check your answers.

- a)  $(+4) + (+3) + (+7)$
- b)  $(-5) + (-7) + (-4)$
- c)  $(+6) + (-9) + (+5)$
- d)  $(-6) + (+3) + (-8) + (+2)$

21. Copy and complete each statement.

- a)  $(+4) + (\blacksquare) = +9$
- b)  $(\blacksquare) + (+4) = -2$
- c)  $(-2) + (\blacksquare) = -7$
- d)  $+6 = (\blacksquare) + (-3)$
- e)  $(+8) + (\blacksquare) = +2$
- f)  $-1 = (-5) + (\blacksquare)$

22. The integer  $-9$  can be expressed as the sum of three consecutive integers.
- $$-9 = (-2) + (-3) + (-4)$$
- a) Identify five other integers that can be expressed as the sum of three consecutive integers.
- b) Suggest a rule that describes all the integers that can be expressed as the sum of three consecutive integers.

### Literacy Link

#### Consecutive Numbers

Consecutive numbers follow one after another in order. For example, 1, 2, 3, 4 are consecutive whole numbers.

23. Hing went out for a walk one morning. She carried her money in a purse. During her walk, she spent \$20 on a DVD movie and bought a sandwich for \$5. When Hing got home in the afternoon, she found \$3 in her room. She put the \$3 in her purse and went back out to buy a magazine, which cost \$4. When she got home again, she had \$2 left in her purse. How much money was in Hing's purse when she first went out that morning?

## MATH LINK

A hockey player's plus/minus (+/–) score in a game is calculated as follows.

- The player scores +1 if he or she is on the ice when the player's team scores an even-strength or short-handed goal.
- The player scores –1 if he or she is on the ice when the opposing team scores an even-strength or short-handed goal.
- Power-play goals and penalty shots are not counted.

Canadian Sidney Crosby began playing professionally in the NHL at the age of 18. The table shows Sidney's plus/minus scores in some early games in his NHL rookie season with the Pittsburgh Penguins.

What was Sidney's total plus/minus score in the following games?

Opposing Team	Sidney's +/– Score
Boston Bruins	0
New Jersey Devils	–1
Carolina Hurricanes	–1
Boston Bruins	+2
New Jersey Devils	–2
Boston Bruins	+1
Carolina Hurricanes	–1
New Jersey Devils	–2

- a) the two games against the Carolina Hurricanes
- b) the three games against the Boston Bruins
- c) the three games against the New Jersey Devils
- d) all the games shown in the table

### WWW Web Link

Canada has produced many famous hockey players. To learn more about famous Canadian hockey players, go to [www.mathlinks7.ca](http://www.mathlinks7.ca) and follow the links.

## 9.3

## Explore Integer Subtraction

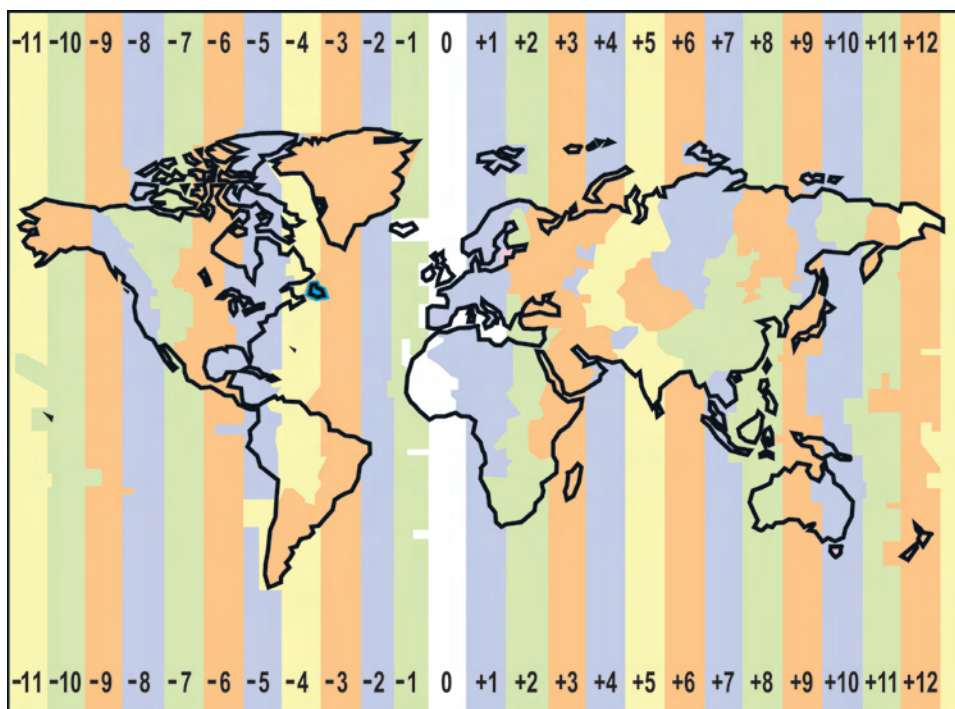
## Focus on...

After this lesson,  
you will be able to...

- subtract integers using integer chips

## Did You Know?

The system of 24 time zones was proposed by a Canadian engineer, Sir Sandford Fleming. He suggested that the time should be the same everywhere in one zone. His idea was adopted in 1884. Before that, individual cities used the position of the sun in the sky to set their own local time.



The map shows how the world is divided into 24 time zones. The zones are labelled using integers. A move from one zone to the next means a time change of one hour. The zone labelled 0 includes London, England. A positive integer shows how many hours the time is ahead of the time in London. A negative integer shows how many hours the time is behind the time in London.

Rome is in time zone +1. Winnipeg is in time zone  $-6$ . Count time zones to find out how many hours Rome is ahead of Winnipeg. Describe the relationship between this time difference and the integers +1 and  $-6$ .

## Explore the Math

## Materials

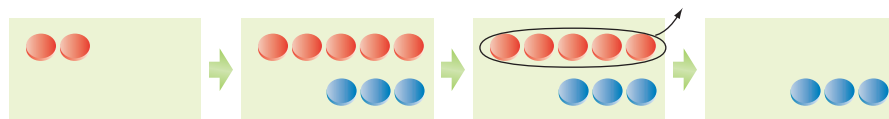
- red and blue integer chips

## How can you use integer chips to subtract two integers?

1. a) Use red chips to model  $+5$ .  
b) Take two of the chips away. Use your observations to copy and complete the subtraction statement  $(+5) - (+2) = \blacksquare$ .  
c) Use chips to model  $(+7) - (+4)$ . Copy and complete the subtraction statement  $(+7) - (+4) = \blacksquare$ .



2. a) Use blue chips to model  $-6$ .  
 b) Take four of the chips away. Use your observations to copy and complete the subtraction statement  $(-6) - (-4) = \blacksquare$ .  
 c) Use chips to model  $(-5) - (-2)$ . Copy and complete the subtraction statement  $(-5) - (-2) = \blacksquare$ .
3. Is it possible to complete the following subtraction statements using the methods from #1 and #2? Explain.
- a)  $(+2) - (+5) = \blacksquare$   
 b)  $(-1) - (-3) = \blacksquare$
4. a) The diagrams show a way to complete the subtraction in #3a) by using zero pairs. Describe the method. Write the subtraction statement.



- b) The diagrams show a way to complete the subtraction in #3b) by using zero pairs. Describe the method. Write the subtraction statement.



5. Model each subtraction. Copy and complete each subtraction statement.
- a)  $(+3) - (+7) = \blacksquare$   
 b)  $(-3) - (-6) = \blacksquare$
6. Use integer chips to model each subtraction. Describe and explain your method in each case. Copy and complete each subtraction statement.
- a)  $(+3) - (-5) = \blacksquare$   
 b)  $(-5) - (+3) = \blacksquare$

## Reflect on Your Findings

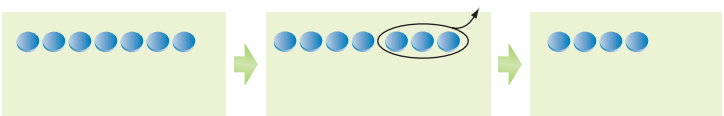
7. How can you use integer chips to subtract two integers? In your description, state when you use zero pairs. How do you determine the number of zero pairs to use?

### Example 1: Subtract Integers Using Integer Chips

Determine each difference using integer chips.  
Copy and complete each subtraction statement.

- a)  $(-7) - (-3) = \blacksquare$       b)  $(+3) - (-4) = \blacksquare$   
c)  $(-6) - (+2) = \blacksquare$       d)  $(-2) - (-5) = \blacksquare$

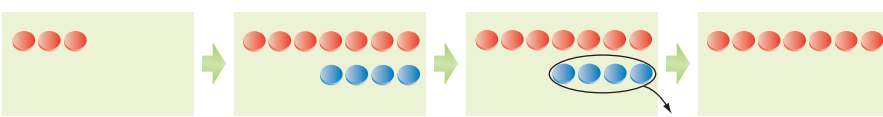
#### Solution

a) 

The difference is  $-4$ .  
The subtraction statement is  $(-7) - (-3) = -4$ .

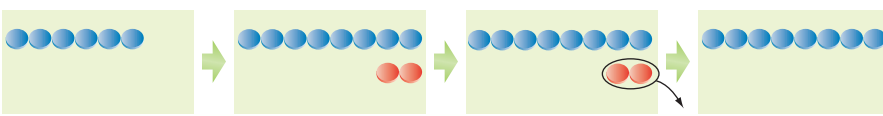
To subtract  $-3$ , remove 3 blue chips.

There are 4 blue chips left.

b) 

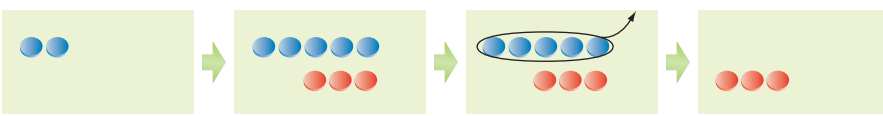
The difference is  $+7$ .  
The subtraction statement is  $(+3) - (-4) = +7$ .

Add 4 zero pairs, so that there are 4 blue chips to remove.

c) 

The difference is  $-8$ .  
The subtraction statement is  $(-6) - (+2) = -8$ .

Add 2 zero pairs, so that there are 2 red chips to remove.

d) 

The difference is  $+3$ .  
The subtraction statement is  $(-2) - (-5) = +3$ .

Add 3 zero pairs, so that there are 5 blue chips to remove.

#### Show You Know

Determine each difference using integer chips. Copy and complete each subtraction statement.

- a)  $(+8) - (+2) = \blacksquare$       b)  $(-1) - (+2) = \blacksquare$   
c)  $(+4) - (-5) = \blacksquare$       d)  $(+4) - (+6) = \blacksquare$

## Example 2: Apply Integer Subtraction

On a typical April day in Whitehorse, Yukon Territory, the temperature is  $-5^{\circ}\text{C}$  in the morning and  $+6^{\circ}\text{C}$  in the afternoon. Determine and explain the difference between these temperatures.

### Solution

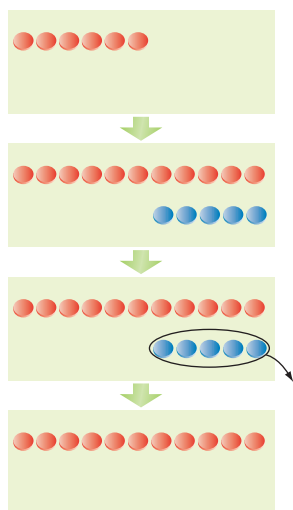
There are two ways to determine the difference between  $-5^{\circ}\text{C}$  and  $+6^{\circ}\text{C}$ .

Use either of the following methods.

#### Method 1: Higher Minus Lower

Subtract the morning temperature from the afternoon temperature.

Subtract  $(+6) - (-5)$ .



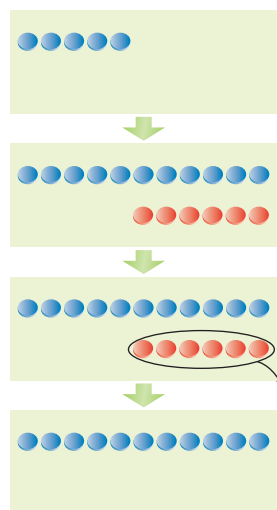
$$(+6) - (-5) = +11$$

The afternoon temperature is  $11^{\circ}\text{C}$  higher than the morning temperature.

#### Method 2: Lower Minus Higher

Subtract the afternoon temperature from the morning temperature.

Subtract  $(-5) - (+6)$ .



$$(-5) - (+6) = -11$$

The morning temperature is  $11^{\circ}\text{C}$  lower than the afternoon temperature.

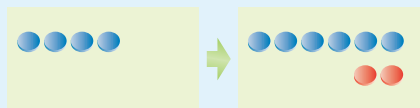
These two sentences agree. Either statement can lead you to the other statement.

## Key Ideas

- You can use integer chips to represent integer subtraction.
- You can model subtraction by removing integer chips of the appropriate colour.
- Add zero pairs if there are not enough chips of one colour to remove.
- There are two differences between two different integers. For example, the differences between  $+1$  and  $-2$  are  $+3$  and  $-3$ .  
 $(+1) - (-2) = +3$        $(-2) - (+1) = -3$

## Communicate the Ideas

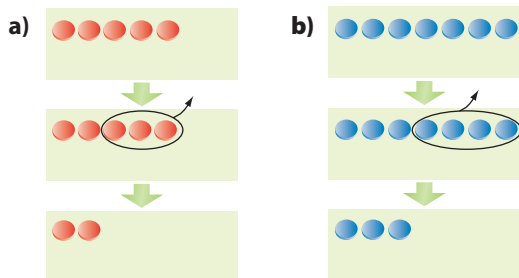
- To model  $(+6) - (+2)$ , Li starts with 6 red chips. What should she do next?
- To model  $(-4) - (-6)$ , Ajani starts with 4 blue chips. He then adds 2 zero pairs.
  - What else should he do to complete the subtraction?
  - Explain why he added 2 zero pairs.
- Explain why adding any number of zero pairs does not change the value represented by a group of integer chips.
- Mario said that the difference between  $+4$  and  $-3$  is  $-7$ . Do you agree? Explain.



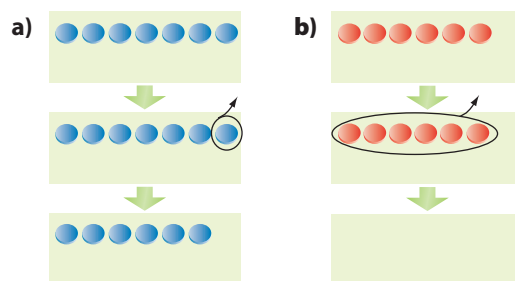
## Practise

For help with #5 to #10, refer to Example 1 on page 325.

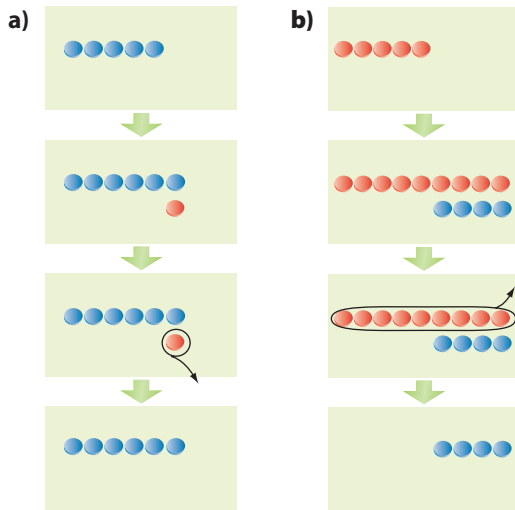
- What subtraction statement does each set of diagrams represent?



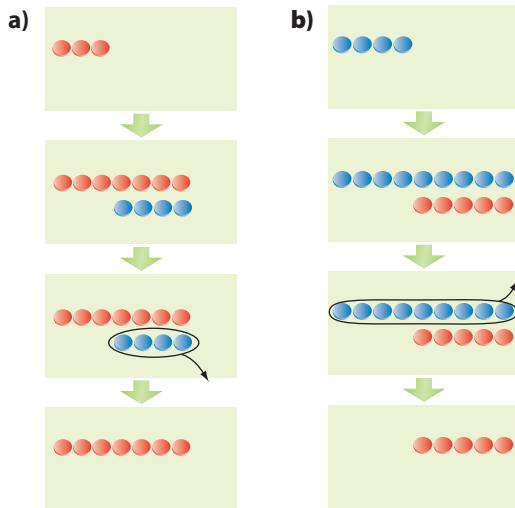
- What subtraction statement does each set of diagrams represent?



7. What subtraction statement does each set of diagrams represent?



8. What subtraction statement does each set of diagrams represent?



9. Determine each difference using integer chips.

- a)  $(+6) - (+4)$
- b)  $(+5) - (-2)$
- c)  $(-6) - (+6)$
- d)  $(-4) - (-7)$

10. Subtract using integer chips.

- a)  $(-5) - (-1)$
- b)  $(+4) - (-2)$
- c)  $(-7) - (+4)$
- d)  $(+3) - (+8)$

*For help with #11 to #16, refer to Example 2 on page 326.*

11. Determine both differences between each pair of integers.

- a)  $+1, +3$
- b)  $+2, -2$
- c)  $0, +4$
- d)  $-9, -7$

12. What are the two differences between each pair of integers?

- a)  $+5, +1$
- b)  $0, -5$
- c)  $-3, +1$
- d)  $-6, -8$

### Apply

13. Determine and explain the time difference between each pair of cities.

- a) Perth, Australia (time zone  $+8$ ); Bermuda (time zone  $-4$ )
- b) Lima, Peru (time zone  $-5$ ); Calgary, Alberta (time zone  $-7$ )
- c) Honolulu, Hawaii (time zone  $-10$ ); Lagos, Nigeria (time zone  $+1$ )
- d) Dawson, Yukon Territory (time zone  $-8$ ); Halifax, Nova Scotia (time zone  $-4$ )

14. On a typical October day in Iqaluit, Nunavut, the temperature is  $-8^{\circ}\text{C}$  in the morning and  $-2^{\circ}\text{C}$  in the afternoon. Determine and explain the difference between these temperatures.

15. Use a subtraction of two integers to complete the following.

- Jamal scored 20 points in a basketball game on Tuesday. He scored 16 points in a game on Wednesday. How many more points did he score on Tuesday than on Wednesday?
- The temperature was  $+3^{\circ}\text{C}$ , and then it fell to  $-2^{\circ}\text{C}$ . What was the temperature decrease?
- Sarah owed Paola \$10. Sarah gave Paola \$4. How much did Sarah still owe?

16. Ariel entered an office building. He rode an elevator up 12 floors from the street level. He then rode down to a parking level that was 3 floors below street level. How many floors did he ride down?



17. Suppose you subtract any non-zero integer from zero. How is the difference related to the original non-zero integer?

18. a) Copy and complete the table.

$(+3) - (+2) = \blacksquare$	$(+2) - (+3) = \blacksquare$
$(+4) - (-1) = \blacksquare$	$(-1) - (+4) = \blacksquare$
$(-3) - (+5) = \blacksquare$	$(+5) - (-3) = \blacksquare$
$(-2) - (-7) = \blacksquare$	$(-7) - (-2) = \blacksquare$

- Compare the two differences on each row of the completed table. How are the two differences between two unequal integers related?
- If you are asked to subtract  $(-4) - (+3)$ , can you subtract the integers in either order? Explain.

### Extend

19. Use integer chips to calculate the following.

- $(+9) - (+2) - (+4)$
- $(-6) - (-4) - (-3)$
- $(+3) - (-1) - (+4)$
- $(-4) - (-2) - (+3)$

20. Copy and complete each subtraction statement.

- $(\blacksquare) - (+1) = +4$
- $(+2) - (\blacksquare) = -3$
- $(-3) - (\blacksquare) = -8$
- $(\blacksquare) - (-4) = 0$

21. The time zone map on page 323 is simplified. Some places have time zones that are not described using integers. For example, Newfoundland is in time zone  $-3\frac{1}{2}$ .

- How many hours is the time in Newfoundland behind the time in London, England?
- How many hours is the time in Newfoundland ahead of the time in Victoria, British Columbia (time zone  $-8$ )? Explain your reasoning.

# 9.4

## Subtract Integers

### Focus on...

After this lesson, you will be able to...

- use addition to subtract integers

The table shows how times during a rocket launch can be described using integers.

Announcement	Meaning	Integer Representation
T minus five seconds	five seconds before liftoff	-5
T plus ten seconds	ten seconds after liftoff	+10

Use integer chips to find the time that passes from T minus five seconds to T plus ten seconds. Would using integer chips be a good way to find the time that passes from T minus 89 seconds to T plus 75 seconds? Explain.



### Explore the Math

#### Materials

- red and blue integer chips

#### How can you use addition to subtract two integers?

1. Copy the table. Use integer chips to help you complete each subtraction statement.

$(+4) - (+3) = \blacksquare$	$(+4) + (-3) = \blacksquare$
$(-5) - (-3) = \blacksquare$	$(-5) + (+3) = \blacksquare$
$(-3) - (+1) = \blacksquare$	$(-3) + (-1) = \blacksquare$
$0 - (+2) = \blacksquare$	$0 + (-2) = \blacksquare$
$(+2) - (-4) = \blacksquare$	$(+2) + (+4) = \blacksquare$
$(-4) - (-5) = \blacksquare$	$(-4) + (+5) = \blacksquare$

2. Compare the two statements on each row of the completed table.  
What pattern do you see?

### Reflect on Your Findings

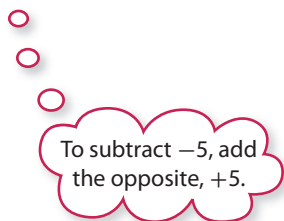
3. a) Describe how you can use addition to subtract two integers.  
b) Test your method on four subtractions of your own.  
Modify your method, if necessary.

### Example 1: Use Addition to Subtract Integers

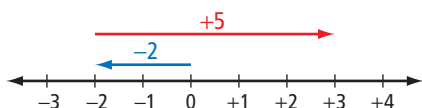
Subtract  $(-2) - (-5)$ .

#### Solution

$$(-2) - (-5) = (-2) + (+5)$$



Use a number line to add  $(-2) + (+5)$ .



The sum is  $(-2) + (+5) = +3$ .

So the difference is  $(-2) - (-5) = +3$ .

### Show You Know

Use addition to complete each subtraction.  
Copy and complete each subtraction statement.

- a)  $(+3) - (+6)$   
b)  $(-2) - (+3)$   
c)  $(-4) - (-8)$



### Did You Know?

The Caspian Sea is the largest lake in the world. The Romans called it a sea because the water is salty.

## Example 2: Calculate and Explain a Difference

The elevation of a lake is the height of its surface from sea level. The elevation of the Caspian Sea is 28 m below sea level. The elevation of Lake Winnipeg is 217 m above sea level. Calculate and explain the difference between these elevations.

### Solution

The elevation of the Caspian Sea is  $-28$  m. The elevation of Lake Winnipeg is  $+217$  m. There are two ways to determine the difference.

#### Method 1: Higher Minus Lower

Subtract the elevation of the Caspian Sea from the elevation of Lake Winnipeg.

Subtract  $(+217) - (-28)$ .

Add the opposite.

$$\begin{aligned} & (+217) - (-28) \\ &= (+217) + (+28) \end{aligned}$$

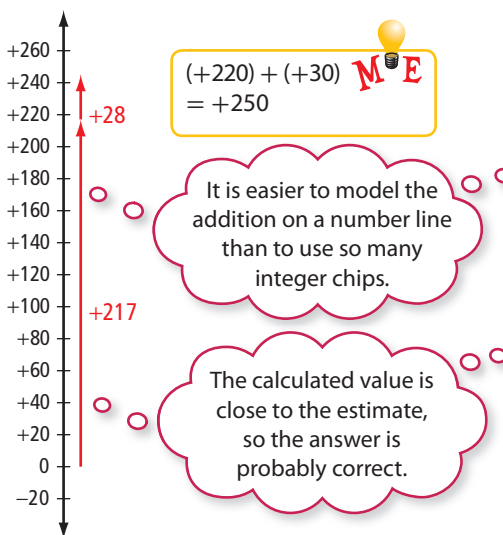
#### Method 2: Lower Minus Higher

Subtract the elevation of Lake Winnipeg from the elevation of the Caspian Sea.

Subtract  $(-28) - (+217)$ .

Add the opposite.

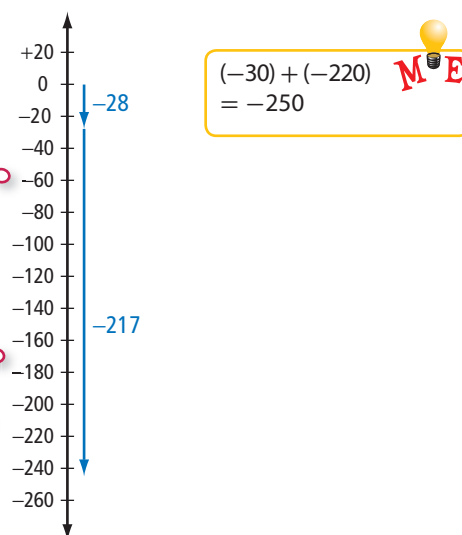
$$\begin{aligned} & (-28) - (+217) \\ &= (-28) + (-217) \end{aligned}$$



$$(+217) + (+28) = +245$$

The difference is  $+245$  m.

The surface of Lake Winnipeg is 245 m higher than the surface of the Caspian Sea.



$$(-28) + (-217) = -245$$

The difference is  $-245$  m.

The surface of the Caspian Sea is 245 m lower than the surface of Lake Winnipeg.

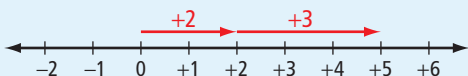
These two sentences agree. Either statement can lead you to the other statement.

## Key Ideas

- You can subtract an integer by adding the opposite.
- You can represent integer subtraction by adding the opposite on a vertical or horizontal number line.

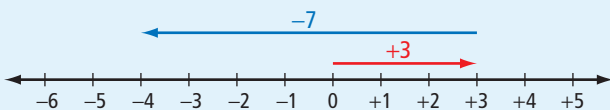
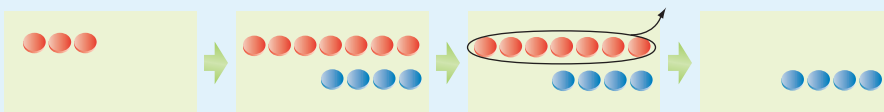
### Communicate the Ideas

1. Mariko drew this diagram to show  $(+2) - (-3)$ . Explain her thinking.



2. Tom said that he could evaluate  $(-3) - (-4)$  by representing  $(+3) + (-4)$  on a number line. Was Tom's thinking correct? Explain.

3. The diagrams show two methods for modelling  $(+3) - (+7)$ .



- a) Do you prefer one method over the other for modelling  $(+3) - (+7)$ ? Explain.
- b) Would you prefer one method over the other for modelling  $(+309) - (+723)$ ? Explain.
4. A difference between two integers is  $+4$ . What is the other difference between the same two integers? How do you know?

### Practise

For help with #5 to #10, refer to Example 1 on page 331.

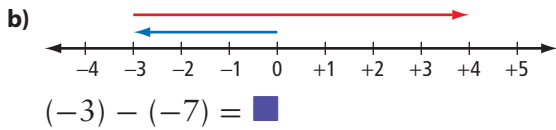
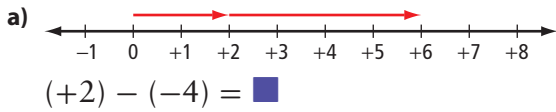
5. Copy and complete each statement.

- a)  $(+3) - (+4) = (+3) + (\blacksquare)$
- b)  $(-1) - (-10) = (-1) + (\blacksquare)$
- c)  $(-4) - (+5) = (-4) + (\blacksquare)$

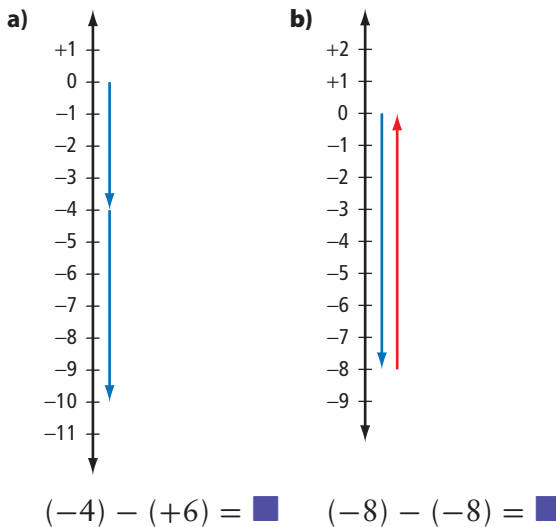
6. Copy and complete each statement.

- a)  $(-7) - (-6) = (-7) + (\blacksquare)$
- b)  $(+6) - (-3) = (+6) + (\blacksquare)$
- c)  $(-9) - (+9) = (-9) + (\blacksquare)$

7. What addition does each diagram represent? Copy and complete the given subtraction statement.



8. What addition does each diagram represent? Copy and complete the given subtraction statement.



9. Subtract.

- a)  $(+3) - (+9)$
- b)  $(-5) - (-10)$
- c)  $(-6) - (+7)$
- d)  $(+4) - (-4)$

10. Subtract.

- a)  $(+9) - (-1)$
- b)  $(-2) - (+5)$
- c)  $(+6) - (+10)$
- d)  $(-1) - (-3)$

For help with #11 to #15, refer to Example 2 on page 332.

11. Determine both differences between each pair of integers.

- a)  $+2, +4$
- b)  $+1, -1$
- c)  $0, +5$

12. What are the two differences between each pair of integers?

- a)  $0, -2$
- b)  $-4, +2$
- c)  $-9, -8$

### Apply

13. The highest land in Saskatchewan is Cypress Hills, at 1468 m above sea level. The lowest land in Saskatchewan is the Lake Athabasca shoreline, at 65 m above sea level. Estimate and calculate the difference between these elevations. Explain your answer.

14. The lowest temperature recorded in Canada was  $-63^{\circ}\text{C}$  at Snag, Yukon Territory. The highest temperature recorded in Canada was  $+45^{\circ}\text{C}$  at Midale, Saskatchewan. What is the difference between these temperatures? Explain your answer.

15. What is the time difference between T minus 44 seconds and T minus 12 seconds for a rocket launch? Explain your answer.

16. The surface of Great Slave Lake is 156 m above sea level. At its deepest point, the bottom of Great Slave Lake is 458 m below sea level. Estimate and calculate the depth of the lake.

17. The element mercury is a silver-coloured liquid at room temperature. The melting point of mercury is  $-39^{\circ}\text{C}$ . The boiling point of mercury is  $357^{\circ}\text{C}$ . How many degrees is the boiling point above the melting point?

18. Suppose a friend knows how to subtract positive integers but has never subtracted negative integers. How could you use the following pattern to show your friend how to subtract  $-2$  from  $+3$ ?

$$\begin{aligned} (+3) - (+3) &= 0 \\ (+3) - (+2) &= +1 \\ (+3) - (+1) &= +2 \\ (+3) - 0 &= +3 \\ (+3) - (-1) &= \blacksquare \\ (+3) - (-2) &= \blacksquare \end{aligned}$$

19. a) Copy and complete each subtraction statement.

$$\begin{aligned} (+1) - (-1) &= \blacksquare \\ (+2) - (-2) &= \blacksquare \\ (+3) - (-3) &= \blacksquare \\ (+4) - (-4) &= \blacksquare \end{aligned}$$

- b) Describe and explain the pattern.  
c) Use the pattern to determine the difference  $(+387) - (-387)$ .
20. What is the length of the line segment joining each pair of points?
- $A(2, 1)$  and  $B(2, 8)$
  - $X(4, -3)$  and  $Y(-5, -3)$
  - $C(-10, 0)$  and  $D(-1, 0)$
  - $P(-4, -2)$  and  $Q(-4, 3)$

21. The vertices of square WXYZ are  $W(-2, -1)$ ,  $X(-2, -5)$ ,  $Y(-6, -5)$ , and  $Z(-6, -1)$ . Determine the perimeter and the area of the square.

22. a) Copy and complete the pattern.

$$\begin{aligned} (+3) - (-2) &= \blacksquare \\ (+2) - (-1) &= \blacksquare \\ (+1) - (0) &= \blacksquare \\ (0) - (+1) &= \blacksquare \end{aligned}$$

- b) Describe and explain the pattern.  
c) Predict the next three lines of the pattern.

### Extend

23. Copy and complete each subtraction statement.

$$\begin{aligned} \text{a) } (+8) - (\blacksquare) &= +3 \\ \text{b) } (\blacksquare) - (+4) &= -5 \\ \text{c) } (-2) - (\blacksquare) &= +7 \\ \text{d) } -6 &= (\blacksquare) - (-1) \\ \text{e) } (\blacksquare) - (+2) &= 0 \\ \text{f) } -2 &= (+5) - (\blacksquare) \end{aligned}$$

24. A difference between two integers is  $-8$ . One of the integers is  $+5$ . What are possible values of the other integer? Show how you know.

## MATH LINK

Canada dominated women's hockey by winning seven of the first eight world championships. The table shows the plus/minus scores of some Canadian players in a game against Finland.

- How much higher was Cassie's score than Caroline's?
- How much lower was Geraldine's score than Jennifer's?
- Determine and explain the greatest difference between two scores in the table.

Player	Plus/Minus Score
Jennifer Botterill	0
Cassie Campbell	+2
Geraldine Heaney	-2
Caroline Ouellette	-1
Cheryl Pounder	+3
Hayley Wickenheiser	+1

# 9.5

## Apply Integer Operations

### Focus on...

After this lesson, you will be able to...

- decide when to add and subtract integers in solving problems

Michelle Wie began playing golf at the age of 4. She became a leading amateur golfer in her early teen years. Michelle played her first professional tournament at age 16, while still in high school.



The number of strokes an expert golfer should take to play a hole is called par. The table shows how integers are used to compare a golfer's score with par.

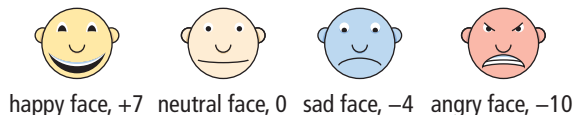
Golf Term	Number of Strokes Above or Below Par
Double Bogey	+2
Bogey	+1
Par	0
Birdie	-1
Eagle	-2

On the same hole, Rosa made an eagle, and Samantha made a bogey. How many more strokes did Samantha take than Rosa?

### Explore the Math

#### How can you decide when to add integers and when to subtract integers?

In this activity, people are represented by different faces. Each face has an integer value.



A group of people with a total value greater than zero is a productive group.

- Decide if each of the following is a productive group.

Show your reasoning.

a)

b)

c)

- What is the least number of happy faces you need to add to each of the following to make a productive group? Show your reasoning.

a)

b)

c)

### Web Link

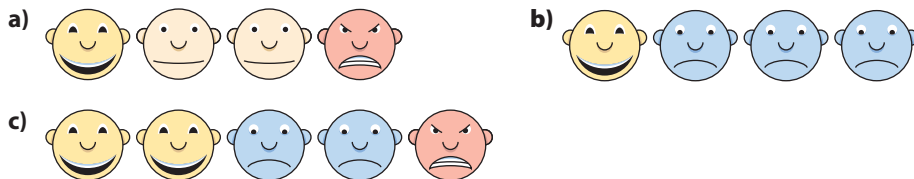
To learn more about the words used in golf, go to [www.mathlinks7.ca](http://www.mathlinks7.ca) and follow the links. What does an "albatross" mean in golf? What integer would you use to represent an albatross?

### Materials

#### Optional

- red and blue integer chips

3. What is the least number of faces you need to remove from each of the following to make a productive group? Show your reasoning.



4. Suppose you change a group in each of the following ways. What happens to the total value of the group? Show your reasoning.

- Replace a neutral face by a sad face.
- Replace an angry face by a happy face.
- Replace a sad face by an angry face.

## Reflect on Your Findings

5. How did you decide when to use addition and when to use subtraction in #1 to #4? Discuss your ideas with your classmates.

### Example: Apply Integer Addition and Subtraction

One February morning in Saskatoon, the air temperature was  $-15^{\circ}\text{C}$ . The wind speed was 20 km/h. By mid-afternoon, the air temperature had increased by  $10^{\circ}\text{C}$ . The wind speed had decreased by 5 km/h. Determine the change in the wind chill from the morning to the afternoon.

#### Solution

Determine the change in the wind chill.

Use a wind chill chart to find the wind chill values in the morning and in the afternoon.

Then subtract to find the change.

Wind Chill Chart

Wind Speed (km/h)	Air Temperature ( $^{\circ}\text{C}$ )							
	5	0	-5	-10	-15	-20	-25	-30
5	4	-2	-7	-13	-19	-24	-30	-36
10	3	-3	-9	-15	-21	-27	-33	-39
15	2	-4	-11	-17	-23	-29	-35	-41
20	1	-5	-12	-18	-24	-31	-37	-43
25	1	-6	-12	-19	-25	-32	-38	-45
30	0	-7	-13	-20	-26	-33	-39	-46
35	0	-7	-14	-20	-27	-33	-40	-47
40	-1	-7	-14	-21	-27	-34	-41	-48
45	-1	-8	-15	-21	-28	-35	-42	-48
50	-1	-8	-15	-22	-29	-35	-42	-49
55	-2	-9	-15	-22	-29	-36	-43	-50
60	-2	-9	-16	-23	-30	-37	-43	-50

#### Did You Know?

Suppose you are outside looking at a thermometer, which reads  $-10^{\circ}\text{C}$ . The wind is blowing at 30 km/h. You feel as cold as you would at  $-20^{\circ}\text{C}$  with no wind. Weather forecasters say that "the wind chill is  $-20$ ."

Understand

Plan

**Do It!****Strategies****Use a Table**

Refer to page xvii.

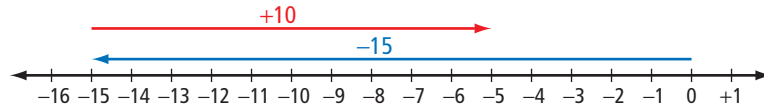
**WWW Web Link**

To learn more about wind chill, go to [www.mathlinks7.ca](http://www.mathlinks7.ca) and follow the links. Choose an air temperature and a wind speed. Then use the Environment Canada wind chill calculator to determine the wind chill.

Use the chart to find the wind chill in the morning. Read along the row from 20 km/h until you reach the column headed  $-15^{\circ}\text{C}$ .

The wind chill in the morning was  $-24$ .

To find the wind chill in the afternoon, you need the air temperature and the wind speed. The air temperature increased by  $10^{\circ}\text{C}$  from  $-15^{\circ}\text{C}$ . The air temperature in the afternoon can be represented by  $(-15) + (+10)$ .

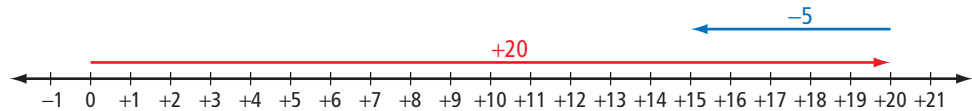


$$(-15) + (+10) = -5$$

The air temperature in the afternoon was  $-5^{\circ}\text{C}$ .

The wind speed decreased by 5 km/h from 20 km/h.

The wind speed can be represented by  $(+20) + (-5)$ .



$$(+20) + (-5) = +15$$

The wind speed in the afternoon was 15 km/h.

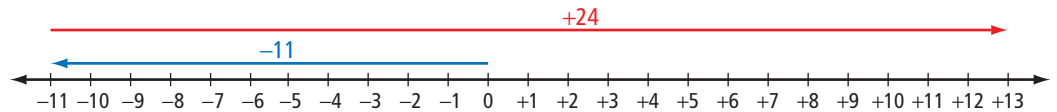
From the wind chill chart, the wind chill in the afternoon was  $-11$ .

Determine the change in the wind chill from  $-24$  to  $-11$ .

The change can be represented by  $(-11) - (-24)$ .

Add the opposite on a number line.

$$(-11) - (-24) = (-11) + (+24)$$



$$(-11) + (+24) = +13$$

$$\text{So } (-11) - (-24) = +13.$$

The wind chill in the afternoon was 13 higher than the wind chill in the morning.

The wind chill increased by 13 from the morning to the afternoon.

You could find the other difference instead.

$$(-24) - (-11) = -13$$

This difference shows that the wind chill in the morning was 13 lower than the wind chill in the afternoon.

Both differences,  $+13$  and  $-13$ , lead to the same conclusion.

**Look Back**

Check that you found the wind chill values correctly in the chart.

Check your calculations using integer chips, or drawings of chips.

## Key Ideas

- You may need to decide when to add integers and when to subtract integers.
- Some problems involve both the addition and subtraction of integers.

### Communicate the Ideas

1. To find the temperature change from  $-10^{\circ}\text{C}$  to  $+10^{\circ}\text{C}$ , would you first write an expression for a sum or a difference? Explain.
2. Suppose you are given \$15, and you spend \$7 of it.
  - a) Represent the amount you have left by an integer subtraction. Explain your reasoning.
  - b) Represent the amount you have left by an integer addition. Explain your reasoning.

### Apply

3. One January day in Calgary, a chinook raised the temperature from  $-17^{\circ}\text{C}$  to  $+13^{\circ}\text{C}$ . What was the change in temperature?
4. One April morning in Churchill, Manitoba, the temperature was  $-14^{\circ}\text{C}$ . In the late afternoon, the temperature was  $9^{\circ}\text{C}$  higher. What was the temperature in the late afternoon?
5. Mauna Kea is a mountain in Hawaii. The base of the mountain is 6033 m below the surface of the ocean. The mountain peak is 4170 m above the surface of the ocean. Estimate and calculate the height of the mountain.
6. The highest paved road in British Columbia is Highway 3 in the Kootenay Pass. It has an elevation of 1774 m above the Fraser River. It is 1794 m above the George Massey Tunnel under the Fraser River. What is the elevation of the tunnel?
7. One afternoon in Fort Simpson, Northwest Territories, the air temperature was  $-10^{\circ}\text{C}$  and the wind speed was 25 km/h. The air temperature dropped by  $5^{\circ}\text{C}$  in the evening, but the wind chill did not change. Refer to the wind chill chart on page 337. What was the change in the wind speed from the afternoon to the evening?
8. A company made a profit of \$8 million one year and a loss of \$12 million the next year.
  - a) What was the company's total profit or loss over the two years?
  - b) How much better was the company's result in the first year than in the second year?



9. Describe each pattern, and predict the next three integers.
- $+1, +4, +7, +10, \dots$
  - $+9, +5, +1, -3, \dots$
  - $-11, -9, -7, -5, \dots$
  - $+20, +15, +10, +5, \dots$
10. Two integers have a sum of  $-15$  and differences of  $+7$  and  $-7$ . What are the two integers? Show how you checked your answers.
11. Identify four pairs of integers for which the sum and a difference are equal. Describe any pattern you see in the pairs of integers.
12. a) In her first round as a professional golfer, Michelle Wie made 12 pars, 4 birdies, and 2 bogeys. Refer to the chart of golf terms on page 336. How many strokes above or below par was she that round?
- b) The leader after the first round was Annika Sorenstam, who made 10 pars and 8 birdies. How many more strokes did Michelle take than Annika in the first round?
- c) Par for one round of the course was 72 strokes. How many strokes did Annika and Michelle each take to complete the first round?
13. The integer  $-5$  can be expressed as the sum of two consecutive integers.  
 $-5 = (-2) + (-3)$
- Copy and complete the following by expressing each integer as the sum of two consecutive integers.
 
$$+15 = (\blacksquare) + (\blacksquare)$$

$$-9 = (\blacksquare) + (\blacksquare)$$

$$-1 = (\blacksquare) + (\blacksquare)$$

$$-25 = (\blacksquare) + (\blacksquare)$$

- Which integers cannot be expressed as the sum of two consecutive integers?
- Which integers can be expressed as the difference between two consecutive integers?

### Literacy Link

#### Omitting Positive Signs or Brackets

A positive integer can be written without the positive sign or brackets. For example,  $(+2) + (+4)$  can be written as  $2 + 4$ . A negative integer must include the negative sign. The brackets can be omitted from a negative integer that does not follow an operation symbol. For example,  $(-3) - (-2)$  can be written as  $-3 - (-2)$ .

14. Calculate.
- $3 - 5$
  - $4 + (-3)$
  - $-6 + 2$
  - $4 - (-6)$
  - $-8 - (-8)$
  - $-4 - 7$
15. Create your own problem that involves integer addition or subtraction. Make sure you can solve your problem. Give your problem to a classmate to solve.
- Extend**
16. A bill was defeated in parliament. Of the MPs who voted, 135 voted in favour and 160 voted against.
- Assume that the same MPs will vote again on the same bill. What is the least number of MPs who will need to change their vote for the bill to pass?
  - Assume that the least number of MPs change their vote, and the bill passes. What are the votes in favour and against?

17. a) A cargo plane left Vancouver, British Columbia (time zone  $-8$ ) at 3:00 p.m. on a Monday. The plane flew for 11 h to Paris, France (time zone  $+1$ ). At what local time and on what day did the plane land in Paris?
- b) The plane left Paris 3 h later for the return flight to Vancouver. Strong headwinds slowed the plane, so this flight took 12 h. At what local time and on what day did the plane land in Vancouver?
- c) After some maintenance work, the plane next left Vancouver at 11:00 a.m. on Wednesday for Honolulu, Hawaii (time zone  $-10$ ). The plane landed in Honolulu at 3:00 p.m. local time that day. How long did the flight take?

18. Historical dates can be described using the abbreviations B.C.E. (Before the Common Era) and C.E. (the Common Era). For example, Augustus ruled the Roman Empire from 27 B.C.E. to 14 C.E. One way to find the number of years he ruled is to do the integer subtraction  $14 - (-27)$ , and then subtract 1. Augustus ruled for 40 years.
- a) How many years was it from 18 B.C.E. to 45 C.E.?
- b) What year was 19 years before 5 C.E.?
- c) What year was 38 years after 21 B.C.E.?

### Did You Know?

The number of years from 27 B.C.E. to 14 C.E. was 40, not 41, because there was no year zero. (Imagine an integer number line with  $+1$  and  $-1$  next to each other.)

## Science Link

### Relative Measurements

The table shows the number of hours of sleep that certain creatures need in a day, relative to the sleep that adult humans need.

1. a) How many more hours of sleep does a cheetah need than an African elephant?
- b) How many less hours of sleep does a goat need than a koala?
2. a) Adult humans need to sleep about eight hours a night. Explain why the value shown in the table for adult humans is 0.
- b) Modify the table to show the actual number of hours that each creature needs to sleep in a night. Explain your reasoning.
- c) Repeat both parts of #1 using your modified table. Do the answers change?
3. a) Make your own table using the ages of some people you know, relative to your age. Make sure that you include both positive and negative values in the table.
- b) Write two problems based on your table.
- c) Have a classmate use your table to solve your problems.

Creature	Hours of Sleep Relative to Adult Humans
Koala	+14
Pig	+11
Three-toed Sloth	+6
Cheetah	+4
Chimpanzee	+2
Human	0
Goat	-3
African Elephant	-5

## Key Words

Copy and complete each statement in #1 and #2.

- Examples of opposite integers are +2 and  $\blacksquare$ .
- In a zero pair of integer chips, one chip represents  $\blacksquare$  and the other chip represents  $\blacksquare$ .

## 9.1 Explore Integer Addition, pages 310–315

- What addition statement does each diagram represent?



- Add using integer chips. Copy and complete the addition statement.

a)  $(-5) + (-3) = \blacksquare$

b)  $(+4) + (-4) = \blacksquare$

c)  $(+6) + (-3) = \blacksquare$

d)  $(-9) + (+4) = \blacksquare$

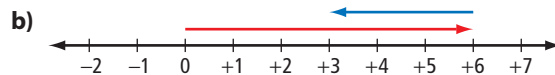
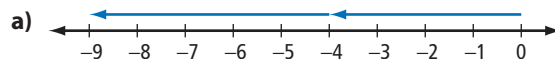
- How can you tell by looking at two integers if their sum is positive, negative, or zero? Use examples to help explain your answer.

- A pelican was flying at a height of 4 m above the ocean. The pelican dove 6 m down and caught a fish. Use the sum of two integers to represent the situation. What is the sum?



## 9.2 Add Integers, pages 316–322

- What addition statement does each diagram represent?



- Add using a number line. Copy and complete the addition statement.

a)  $(-3) + (+3) = \blacksquare$

b)  $(+7) + (-2) = \blacksquare$

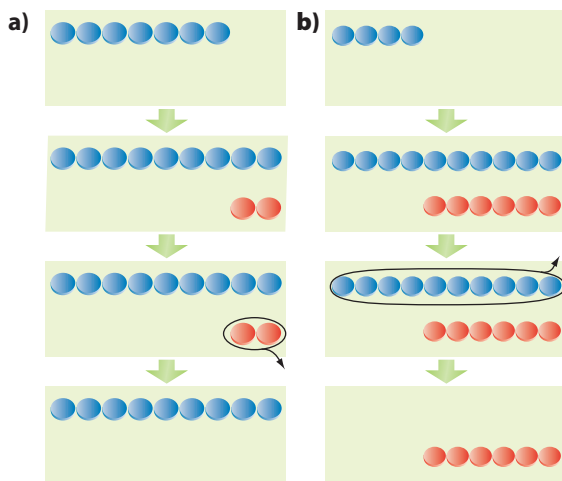
c)  $(-4) + (+12) = \blacksquare$

d)  $(+6) + (-8) = \blacksquare$

- The sum of two integers is  $-9$ . What could the integers be? Give four possible answers.
- The elevation of Death Valley, California, is 86 m below sea level. New Orleans, Louisiana, is 84 m higher than Death Valley. What is the elevation of New Orleans?

### 9.3 Explore Integer Subtraction, pages 323–329

11. What subtraction statement does each set of diagrams represent?



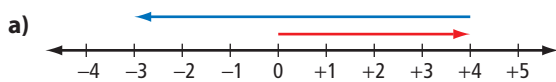
12. Subtract using integer chips. Copy and complete the subtraction statement.

- a)  $(-7) - (-5) = \blacksquare$   
 b)  $(+4) - (-3) = \blacksquare$   
 c)  $(+3) - (+8) = \blacksquare$   
 d)  $(-1) - (+6) = \blacksquare$

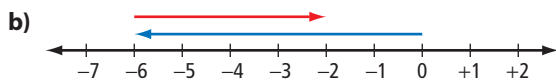
13. How many hours is Hong Kong (time zone +8) ahead of Lethbridge, Alberta (time zone -7)?

### 9.4 Subtract Integers, pages 330–335

14. What addition statement does each diagram represent? Use the diagram to help you copy and complete the given subtraction statement.



$(+4) - (+7) = \blacksquare$



$(-6) - (-4) = \blacksquare$

15. Subtract.

- a)  $(+5) - (+8)$   
 b)  $(-4) - (-6)$   
 c)  $(-2) - (+7)$   
 d)  $(+4) - (-9)$

16. The elevation of the top of Mt. Everest is 8848 m above sea level. The elevation of the Dead Sea is 411 m below sea level. Estimate, calculate, and explain the difference between these elevations.

### 9.5 Apply Integer Operations, pages 336–341

17. Describe each pattern, and predict the next three integers.

- a)  $+3, +9, +15, +21, \dots$   
 b)  $+40, +30, +20, +10, \dots$

18. The table shows Mike Weir's performance when he won the Masters golf tournament.

Round	Number of Strokes Above or Below Par
1	-2
2	-4
3	+3
4	-4

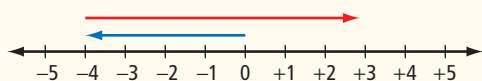
- a) What is the difference in Mike's performance in the third and fourth rounds?  
 b) What is the difference in Mike's performance in the first two rounds?  
 c) How many strokes above or below par was Mike for the whole tournament?  
 d) Par for one round of the course was 72 strokes. How many strokes did Mike take altogether to complete the four rounds?

For #1 to #6, select the correct answer.

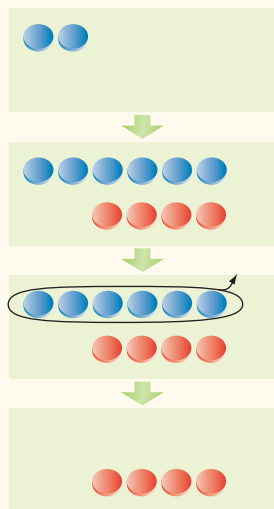
1. Which addition do the integer chips represent?



- A  $(+8) + (+5)$   
 B  $(-5) + (+8)$   
 C  $(-8) + (-5)$   
 D  $(-8) + (+5)$
2. Which addition does the diagram represent?



- A  $(-4) + (+7)$   
 B  $(-7) + (+4)$   
 C  $(+7) + (+4)$   
 D  $(-7) + (-4)$
3. What subtraction statement does the set of diagrams represent?



- A  $(-6) - (-2)$   
 B  $(-2) - (-6)$   
 C  $(-2) - (+4)$   
 D  $(-2) - (+6)$
4. Which expression equals  $(-2) + (+6)$ ?
- A  $(-6) + (+2)$   
 B  $(+6) + (-2)$   
 C  $(+2) + (-6)$   
 D  $(+6) + (+2)$
5. Which expression equals  $(+4) - (-3)$ ?
- A  $(-3) - (+4)$   
 B  $(+4) - (+3)$   
 C  $(+4) + (+3)$   
 D  $(-4) + (-3)$
6. Which expression does not equal 0?
- A  $(-3) + (+3)$   
 B  $(-3) - (-3)$   
 C  $(+3) - (+3)$   
 D  $(-3) - (+3)$

Complete the statements in #7 and #8.

7. One afternoon, the temperature in Edmonton, Alberta, was  $+6^{\circ}\text{C}$ . The temperature then dropped by  $10^{\circ}\text{C}$  to reach the overnight low temperature. The overnight low temperature was  $\blacksquare^{\circ}\text{C}$ .
8. The length of the line segment joining  $(4, -9)$  and  $(4, 8)$  is  $\blacksquare$  units.

Short Answer

9. What is each sum or difference?
- a)  $(-4) + (-5)$       b)  $(+3) + (-8)$   
 c)  $(-9) - (-7)$       d)  $(+6) - (-6)$   
 e)  $(+2) - (+7)$       f)  $(-3) - (-8)$

10. A monkey climbed 6 m down a tree and then climbed 4 m back up.
- Use the sum of two integers to represent the situation.
  - What is the sum?
11. Determine the perimeter and the area of the rectangle with vertices  $C(2, -3)$ ,  $D(-2, -3)$ ,  $E(-2, -6)$ ,  $F(2, -6)$ .
12. The surface of Lake Michigan is 176 m above sea level. The bottom of the lake is 105 m below sea level. Estimate and calculate the depth of the lake.
13. Identify two integers that have a sum of +16 and differences of  $-8$  and  $+8$ .

- subtracting a negative integer from a positive integer
- subtracting a positive integer from a negative integer
- subtracting a negative integer from a negative integer

15. The table shows some temperatures expressed in degrees Celsius and in kelvins.

Situation	Temperature (°C)	Temperature (K)
Water boils	100	373
Water freezes	0	273
Absolute zero	$-273$	0

- Describe the relationship between the temperature values in the two columns.
- Convert 250 K to degrees Celsius.
- Convert  $-100^{\circ}\text{C}$  to kelvins.

### Extended Response

14. In which of the following cases do you always know the sign of the difference? Explain your reasoning and give examples.
- subtracting a positive integer from a positive integer

## WRAP IT UP!

The table shows the plus/minus scores for Jarome Iginla in four of his seasons with the Calgary Flames.

- What was the total of Jarome's plus/minus scores for the following seasons?
  - 2000–2001 and 2001–2002
  - 2002–2003 and 2003–2004
  - all four seasons shown in the table
- Calculate and explain the difference between Jarome's plus/minus scores for the 2002–2003 and 2003–2004 seasons.
- Research the plus/minus scores for a hockey player of your choice. Describe how you would use the scores to measure the hockey player's performance.

Season	Plus/Minus Score
2000–2001	$-2$
2001–2002	$+27$
2002–2003	$-10$
2003–2004	$+21$

# Math Games

## Integer Word Game

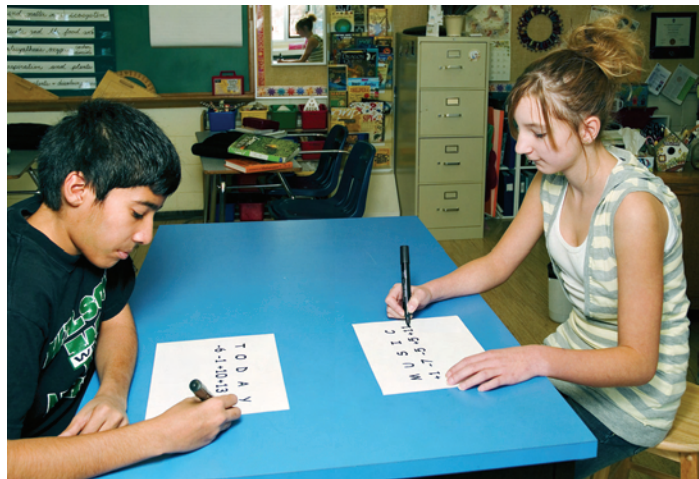
Play this word game in a pair or a group. In this game, each letter of the alphabet is assigned an integer value. The value of a word is the sum of the values of the letters in the word.

A	B	C	D	E	F	G	H	I	J	K	L	M
+13	+12	+11	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12

Work individually to spell words. Then compare your words and values in your pair or group. Each word must be a real word but must not be a proper noun, such as a person's name.

1. Create a three-letter word with
  - a) the greatest value
  - b) the least value
  - c) a value of 0
2. Repeat #1 for five-letter words.
3. Who can create the most five-letter words with a value of 0? Describe the strategies that helped you create them.



# Challenge in Real Life

## Virtual Reality

Some arcade games use virtual reality to boost excitement. The player sits on a simulated vehicle such as a motorcycle, snowmobile, or all-terrain vehicle. The seat moves to imitate the motion of the wild ride the player is watching on the screen.

You be the designer! Create a virtual reality game in which a rider sits on a vehicle and watches a screen as the vehicle moves up and down. The player scores points by driving over the rises and drops without crashing.



- Describe your vehicle and give your game a name.
- Plan a series of between 10 and 30 rises and drops for your game.
  - Choose the height and depth of the vehicle after each move.
  - The starting point of the ride will be shown as zero. Heights above zero are shown as positive integers and depths below zero as negative integers.
  - The player scores points this way: If the vehicle drops from a height of  $+15$  to a depth of  $-5$ , that is a total change of  $-20$ . The player gets 20 points if the vehicle does not crash during the drop.
  - To avoid a crash, the maximum rise is 25 and the maximum drop is  $-25$ .

- Make a table of the heights and depths like the one shown.

Move Number	Height or Depth	Ordered Pair
0	0	(0, 0)
1	+6	(1, +6)
2	-4	(2, -4)

- Show the heights and depths of the ride by plotting the ordered pairs on a coordinate grid.
- Pick the three most extreme rises or drops in your game. Use integers to show the amount of each rise or drop. Use positive integers for rises and negative integers for drops.
- What is the maximum number of points a player can score in your game?

Both positive and negative changes score positive points